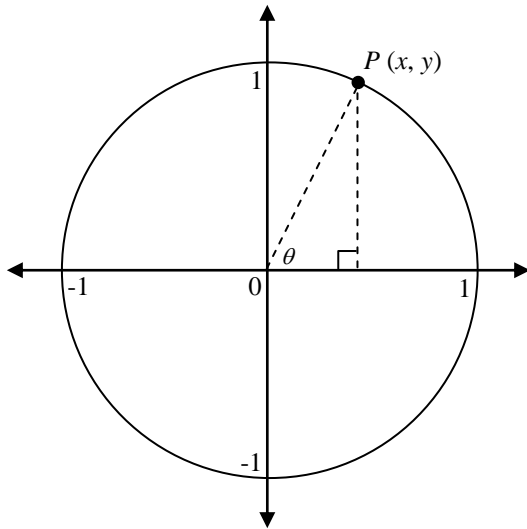


# Trigonometry and the Unit Circle

The following diagram shows a circle of radius 1 (this is called a unit circle). Point  $P$  is used to create a right-angled triangle. The coordinates of  $P$  are  $(x, y)$ .



Level 1 – 2

- Write down the length of the hypotenuse of the triangle.

.....

- Complete the following by writing  $x$  or  $y$ .

i) the length of the base of the triangle is .....

ii) the height of the triangle is .....

- Determine an expression for the value of the following in terms of  $x$  and/or  $y$ . Simplify your answers.

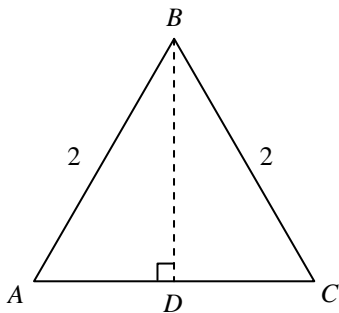
i)  $\cos \theta$  .....

ii)  $\sin \theta$  .....

You should have determined:

The  $x$ -coordinate of  $P$  represents the value of  $\cos \theta$  and the  $y$ -coordinate represents the value of  $\sin \theta$ .

The following diagram represents an equilateral triangle with sides of length 2. The triangle is cut in half.



- Write down the length of  $AD$ . .....

- Write down the size of angle  $DAB$ . .....

- Write down the size of angle  $ABD$ . .....

- Determine the length of  $BD$ . .....

.....

- Show that  $\sin 60 = \frac{\sqrt{3}}{2}$ .

.....

.....

- Show that  $\cos 60 = \frac{1}{2}$ .

.....

.....

10. Show that  $\sin 30 = \frac{1}{2}$ .

.....  
 .....

11. Show that  $\cos 30 = \frac{\sqrt{3}}{2}$ .

.....  
 .....

*Level 3 – 4*

12. Use a different diagram to show that  $\sin 45 = \cos 45 = \frac{\sqrt{2}}{2}$ .

.....  
 .....

13. Use the definitions of sine and cosine to show that  $\tan \theta = \frac{\sin \theta}{\cos \theta}$ .

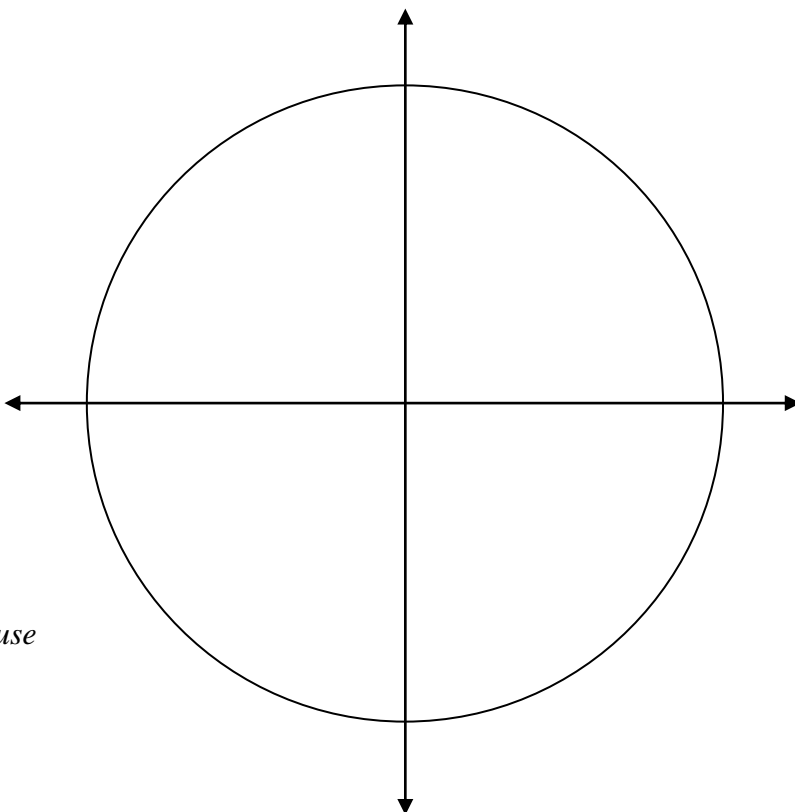
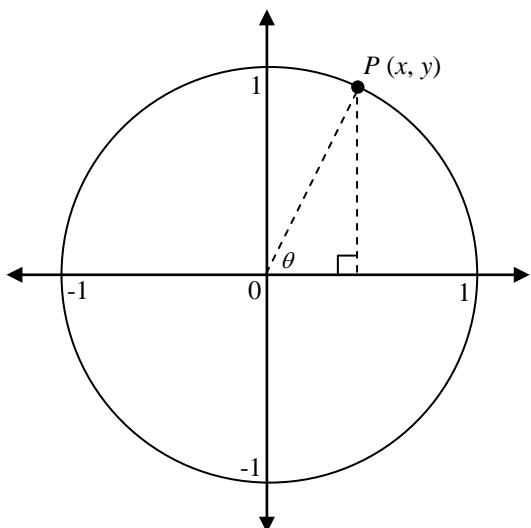
.....  
 .....

14. Use previous results to complete the following table showing the values of  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$  for various values of  $\theta$ . Show any calculations you made in the space on the right.

$\theta$	$\sin \theta$	$\cos \theta$	$\tan \theta$
30			
45			
60			

.....  
 .....

Recall that in our unit circle the  $x$ -coordinate of  $P$  represents the value of  $\cos \theta$  and the  $y$ -coordinate represents the value of  $\sin \theta$ . This applies for any value of  $\theta$ . Use this rule to answer all remaining questions.



The blank unit circle on the right is for you to use to help you answer the next question.

Level 5 – 6

15. Use symmetry to determine the following values. Some values cannot be calculated. In these cases write “undefined”. Negative angles are measured clockwise.

$\cos 90$		$\cos 225$	
$\sin 90$		$\sin 225$	
$\tan 90$		$\tan 225$	
$\cos 120$		$\cos 315$	
$\sin 120$	$\sqrt{3}/2$	$\sin 315$	
$\tan 120$		$\tan 315$	
$\cos 135$	$-\sqrt{2}/2$	$\cos(-120)$	
$\sin 135$		$\sin(-120)$	
$\tan 135$		$\tan(-120)$	
$\cos 150$		$\cos 330$	
$\sin 150$		$\sin 330$	$-1/2$
$\tan 150$		$\tan 330$	
$\cos 180$		$\cos(-60)$	
$\sin 180$		$\sin(-60)$	
$\tan 180$		$\tan(-60)$	

16. Write the following in terms of  $\sin \theta$ ,  $\cos \theta$  or  $\tan \theta$ . Justify each answer.

a)  $\sin(-\theta)$   $-\sin(\theta)$   
.....

.....  
Since the value of  $\sin(\theta)$  represents the y coordinate of  $P$ , and a negative angle  
.....  
means we measure clockwise, we must have  $\sin(-\theta) = -\sin \theta$   
.....

b)  $\cos(-\theta)$  .....  
.....  
.....

c)  $\tan(-\theta)$  .....  
.....  
.....

d)  $\sin(180 - \theta)$  .....  
.....  
.....

e)  $\cos(180 - \theta)$  .....  
.....  
.....

f)  $\tan(180 - \theta)$  .....  
.....  
.....

17. Use the first table to complete the second table to 3 significant figures:

$\theta$	$\sin \theta$	$\cos \theta$	$\theta$	$\sin \theta$	$\cos \theta$
4	0.070	0.998	49	0.755	0.656
13	0.225	0.974	62	0.883	0.469
25	0.423	0.906	71	0.946	0.326
38	0.616	0.788	82	0.990	0.139

$\theta$	$\sin \theta$	$\cos \theta$
109		
167		
184		
335		
-49		
242		
193		
-109		