Solving Quadratic Equations by Completing the Square

Level 1 – 2

1. Expand and simplify the following:

a) $(x+1)^2$ $= (x+1)(x+1) = x^2 + x + x + 1 = x^2 + 2x + 1$

b) $(x+3)^2$ .....

c) $(x+7)^2$ .....

d) $(x-2)^2$ .....

e) $(x-5)^2$ .....

f) $(x-6)^2$ .....

2. Factorise the following, writing your answers in the form $(x+a)^2$.

a) $x^2 + 10x + 25$ $(x+5)^2$

b) $x^2 - 6x + 9$ .....

c) $x^2 + 12x + 36$ .....

d) $x^2 + 8x + 16$ .....

e) $x^2 - 2x + 1$ .....

f) $x^2 - 14x + 49$ .....

3. Look at questions 1 and 2. If $(x+a)^2 = x^2 + bx + c$

a) How does the value of *b* compare to the value of *a*?

.....

b) How does the value of *c* compare to the value of *a*?

.....

Level 3 – 4

4. Fill in the missing values:

a) $(x+8)^2 = x^2 + \ldots\ldots x + 64$

b) $(x - \ldots\ldots)^2 = x^2 - 18x + 81$

c) $(x + \ldots\ldots)^2 = x^2 + 20x + 100$

d) $(x-11)^2 = x^2 - 22x + \ldots\ldots$

e) $(x - \ldots\ldots)^2 = x^2 - 2x + \ldots\ldots$

f) $(x + \ldots\ldots)^2 = x^2 + \ldots\ldots x + 144$

All of the above quadratic expressions are called *perfect square trinomials* because they can be written in the form $(x+a)^2$.

5. Which of the following are perfect square trinomials? (Circle your choices)

a) $x^2 + 18x + 81$

b) $x^2 - x - 2$

c) $x^2 - 30x + 225$

d) $x^2 + 8x + 9$

e) $x^2 - 4x + 7$

f) $x^2 - 16x + 64$

6. Expand and simplify the following:

a) $(x+8)^2 - 3$ $= (x+8)(x+8) - 3 = x^2 + 8x + 8x + 64 - 3 = x^2 + 16x + 61$

b) $(x+2)^2 - 1$ ……

c) $(x-3)^2 - 6$ ……

d) $(x+5)^2 - 4$ ……

e) $(x+1)^2 - 2$ ……

7. Fill in the missing values:

a) $x^2 + 4x + 1 = (x+2)^2 - \ldots\ldots$

b) $x^2 - 6x + 8 = (x-3)^2 - \ldots\ldots$

c) $x^2 - 2x - 2 = (x-1)^2 - \ldots\ldots$

d) $x^2 + 10x + 22 = (x+5)^2 - \ldots\ldots$

e) $x^2 - 8x + 15 = (x - \ldots\ldots)^2 - 1$

f) $x^2 - 6x + 7 = (x - \ldots\ldots)^2 - 2$

g) $x^2 + 12x + 30 = (x + \ldots\ldots)^2 - 6$

h) $x^2 - 10x + 21 = (x - \ldots\ldots)^2 - 4$

i) $x^2 + 8x + 10 = (x + \ldots\ldots)^2 - \ldots\ldots$

j) $x^2 - 4x - 3 = (x - \ldots\ldots)^2 - \ldots\ldots$

k) $x^2 + 16x + 60 = (x + \ldots\ldots)^2 - \ldots\ldots$

l) $x^2 - 10x + 15 = (x - \ldots\ldots)^2 - \ldots\ldots$

Level 5 – 6

A number is in simplest radical form when the number underneath the square root sign is the smallest possible value. For example

$\sqrt{8}$ in simplest radical form is $\sqrt{4\times 2} = \sqrt{4}\times\sqrt{2} = 2\sqrt{2}$

$\sqrt{27}$ in simplest radical form is $\sqrt{9\times 3} = \sqrt{9}\times\sqrt{3} = 3\sqrt{3}$

$\dfrac{2\pm\sqrt{8}}{4}$ in simplest radical form is $\dfrac{2\pm 2\sqrt{2}}{4} = \dfrac{1\pm\sqrt{2}}{2}$

8. Write the following in simplest radical form. Show your working out.

a) $\sqrt{125}$ .....

b) $\sqrt{72}$ .....

c) $\dfrac{3+\sqrt{90}}{6}$ .....

.....

d) $\dfrac{4\pm\sqrt{98}}{5}$ .....

.....

9. Solve the following equations by writing the left side in the form $(x+a)^2+b$. This method is called *completing the square*. Write your answers in the simplest radical form.

a) $x^2+4x+1=0$ $(x+2)^2-3=0$

$\Rightarrow (x+2)^2 = 3$

$\Rightarrow x+2 = \pm\sqrt{3}$

$\Rightarrow x = -2\pm\sqrt{3}$

b) $x^2 + 10x + 20 = 0$ .....

.....

.....

.....

.....

.....

c) $x^2 - 8x + 15 = 0$ .....

.....

.....

.....

.....

.....

d) $x^2 - 2x - 1 = 0$ .....

.....

.....

.....

.....

.....

e) $2x^2 + 6x + 8 = 0$ .....

.....

.....

.....

.....

.....

10. Solve $3x^2 + 10x + 2 = 0$ by completing the square. Write your answer in the simplest radical form.

……

……

……

……

……

……

11. Solve $4x^2 + 5x - 6 = 0$ by completing the square. Write your answer in the simplest radical form.

……

……

……

……

……

……

12. Solve $ax^2 + bx + c = 0$ by completing the square. Show your full working out.

……

……

……

……

……

……

……

……