Radicals

All solutions must be in the simplest radical form i.e. $x = \dfrac{a \pm b\sqrt{c}}{d}$ *where a, b, c and d are integers , c is the lowest possible value, and a, b and d are in the lowest terms. The value of d should be positive.*

Level 1 – 2

1. Simplify:

a) $2\sqrt{3} - 5\sqrt{3}$ ………………………… b) $4\sqrt{10} + \sqrt{10}$ …………………………

c) $\frac{3\sqrt{20}}{2\sqrt{5}}$ ………………………… d) $\left(\frac{1}{2\sqrt{3}}\right)^2$ …………………………

e) $\left(\frac{-\sqrt{5}}{2\sqrt{7}}\right)^2$ ………………………… f) $\left(\sqrt{5}\right)^3$ …………………………

2. Expand and simplify:

a) $\sqrt{3}\left(1 + \sqrt{3}\right)$ …………………………………………………………………………

b) $\sqrt{5}\left(1 - 3\sqrt{5}\right)$ …………………………………………………………………………

c) $\left(2 + \sqrt{7}\right)\left(2 - \sqrt{7}\right)$ …………………………………………………………………………

d) $\left(1 + \sqrt{2}\right)\left(3 + \sqrt{2}\right)$ …………………………………………………………………………

e) $\left(2 + \sqrt{3}\right)^2$ …………………………………………………………………………

3. Write with an integer denominator in the simplest form:

a) $\dfrac{1}{\sqrt{3}}$ ………………………… b) $\dfrac{5\sqrt{2}}{\sqrt{5}}$ …………………………

c) $\dfrac{\sqrt{3}}{\sqrt{12}}$ ………………………… d) $\dfrac{\sqrt{8}}{3\sqrt{2}}$ …………………………

Level 3 – 4

4. Write in simplest radical form:

a) $\dfrac{\sqrt{5}}{3-\sqrt{5}}$ ……………………………………………………………………

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b) $\dfrac{3}{2+\sqrt{3}}$ ……………………………………………………………………

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c) $\dfrac{1}{3+\sqrt{5}}+\dfrac{2}{2-\sqrt{5}}$ ……………………………………………………………………

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5. Determine the values of *x* and *y*:

a) $2+3\sqrt{3}=x+\sqrt{y}$ ……………………………………………………………………

b) $4-2\sqrt{5}=4x-\sqrt{y}$ ……………………………………………………………………

c) $5+3\sqrt{6}=-x+\sqrt{3y}$ ……………………………………………………………………

d) $x+y+\sqrt{x-y}=11+\sqrt{7}$ ……………………………………………………………………

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6. Show that $\sqrt{18}=\sqrt{2}+\sqrt{8}$.

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Level 5 – 6

7. Replace x in the equation $x^2 - 2x - 1 = 0$ with $1+\sqrt{2}$ to show that $x = 1+\sqrt{2}$ is a solution to the equation.

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8. Expand and simplify $\left(1+\sqrt{2}\right)^3$

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9. a) Expand and simplify $(a+\sqrt{b})(a-\sqrt{b})$.

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b) Hence write the following in the form $(a+\sqrt{b})(a-\sqrt{b})$ where both a and b are positive integers less than 6.

i) 23 ……………………………………………………………………………

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ii) 13 ……………………………………………………………………………

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Level 7 – 8

10. Write $\sqrt{27+10\sqrt{2}}$ in the form $a+\sqrt{b}$.

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11. Given that:

$$x=\sqrt{2+\sqrt{3+\sqrt{2+\sqrt{3+...}}}}$$

Show that $x^4-4x^2-x+1=0$.

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