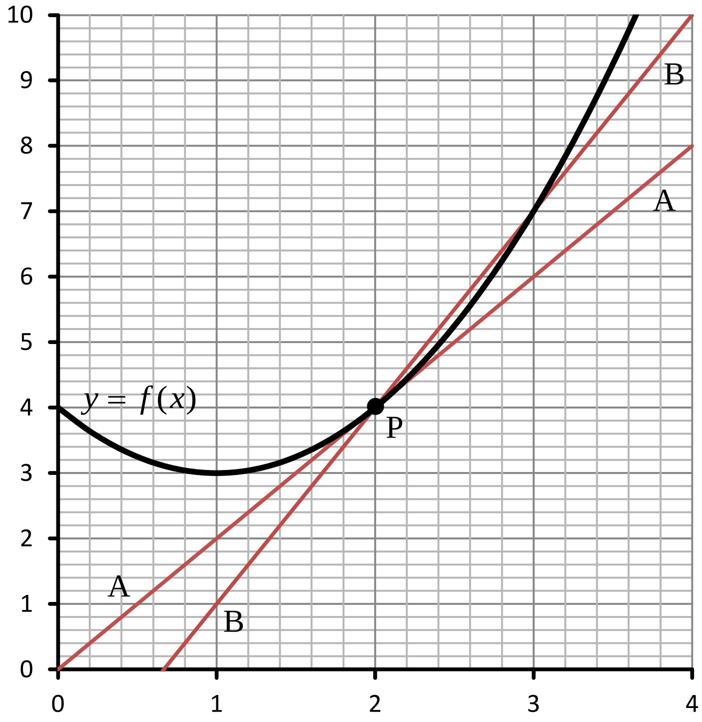
Function Notation Extension - Determining the Gradient of a Curve

The following diagram shows the graph of a function $y = f(x)$.



Line A is called the tangent line at point P. It just touches the curve at point P.

Line B is called a secant line of point P. It intersects the curve at point P and one other point.

Level 1 – 2

1. Identify where the secant line intersects the curve again. Label this point Q.

2. Explain what happens as point Q moves closer and closer to point P.

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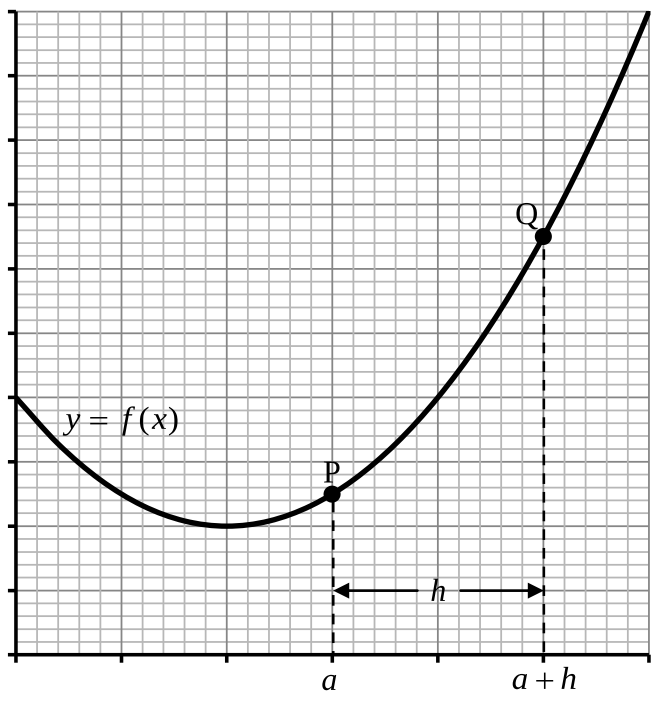
In the following diagram, point P has an x-coordinate of a and point Q has an x coordinate of $a + h$.



3. Draw the secant line passing through points P and Q.

4. Using function notation, write down the y-coordinate of point:

   a) P ……………… b) Q ………………

5. Using function notation, determine an expression for the gradient of the secant line passing through points P and Q. Simplify your answer as much as possible.

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6. Draw the tangent to the curve at point P.

7. Explain what happens as the value of h approaches zero.

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Use the method from the previous page for the remainder of this worksheet.

Level 3 – 4

8. Let $f(x) = x^2$ and $a = 1$

a) Determine an expression in terms of h for the gradient of the secant line passing through points P and Q. Simplify your answer as much as possible.

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b) Hence, determine the gradient of the tangent to the curve $y = x^2$ when $x = 1$.

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9. Let $f(x) = \frac{1}{2}x^2$ and $a = 4$

a) Determine an expression in terms of h for the gradient of the secant line passing through points P and Q. Simplify your answer as much as possible.

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b) Hence, determine the gradient of the tangent to the curve $y = \frac{1}{2}x^2$ when $x = 4$.

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10. Let $f(x) = -2x^2$ and $a = 1$

a) Determine an expression in terms of h for the gradient of the secant line passing through points P and Q. Simplify your answer as much as possible.

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b) Hence, determine the gradient of the tangent to the curve $y = -2x^2$ when $x = 1$.

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11. Let $f(x) = x^2 + x$ and $a = -2$

a) Determine an expression in terms of h for the gradient of the secant line passing through points P and Q. Simplify your answer as much as possible.

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b) Hence, determine the gradient of the tangent to the curve $y = x^2 + x$ when $x = -2$.

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Level 5 – 6

12. Determine the point on the curve of $y = x^3$ where the gradient of the tangent is equal to 6.75.

13. Determine the point on the curve of $y = x^2 + 3x - 1$ where the gradient of the tangent is equal to 7.

Level 7 – 8

14. Determine the point on the curve of $y = \sqrt{x}$ where the gradient of the tangent is equal to $\frac{1}{6}$.

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15. Determine the point on the curve of $y = \frac{3}{x^2}$ where the gradient of the tangent is equal to -2.

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Selected Solutions

5) $\frac{f(a+h)-f(a)}{h}$ 8 a) $2+h$ b) 2 9 a) $4+\frac{1}{2}h$ b) 4 10 a) $-4-2h$ b) -4 11 a) $-3+h$ b) -3

12) $(1.5, 3.375)$, $(-1.5, -3.375)$ 13) $(2,9)$ 14) $(9,3)$ 15) $\left(\sqrt[3]{3}, \sqrt[3]{3}\right)$