# Geometric Sequences

*Level 1 – 2*

1. Which of the following are geometric sequences? Circle your choices.

   a) 1, 3, 9, 27, …           b) 2, 4, 6, 8, …           c) 4, -4, 4, -4, …

   d) 1, 2, 6, 24, …           e) 2, 2, 2, 2, …           f) 80, 78, 75, 71, …

   g) 5, 8, 11, 14, …          h) 32, 16, 8, 4, …         i) 1, 0.1, 0.01, 0.001, …

2. Write down the next three terms of the geometric sequences.

   a) 10, 20, 40, …           ……………………………………………………………………

   b) 120, -60, 30, …         ……………………………………………………………………

   c) 4, 12, 36, …            ……………………………………………………………………

   d) 3, 3, 3, …              ……………………………………………………………………

   e) -4, -8, -16, …          ……………………………………………………………………

3. Calculate the first four terms of the following geometric sequences:

   a) $t_n = 4 \times 2^{n-1}$          ……………………………………………………………………

   b) $t_n = 2t_{n-1}$      $t_1 = 3$          ……………………………………………………………………

   c) $t_n = 2000 \times 0.1^{n-1}$          ……………………………………………………………………

   d) $t_n = 3 \times (-4)^{n-1}$          ……………………………………………………………………

   e) $t_n = -t_{n-1}$      $t_1 = 4$          ……………………………………………………………………

   f) $t_n = t_{n-1} \div 2$      $t_1 = 64$          ……………………………………………………………………

4. Write down one possible missing term in each of the following geometric sequences:

   a) 2, …………, 8           b) 3, …………, 27           c) 1, …………, 16

   d) -1, …………, -100        e) 100, …………, 1          f) 32, …………, 8

   g) 1, …………, 25           h) 2, …………, 0.02         i) -3, …………, -48

*Level 3 – 4*

5. i) Determine an expression for the $n^{th}$ term of the geometric sequences.

ii) Use the expression to determine the $9^{th}$ term.

a) 5, 10, 20, …

i) ……………………………………………………………………………………

ii) ……………………………………………………………………………………

b) 1024, 512, 256, …

i) ……………………………………………………………………………………

ii) ……………………………………………………………………………………

c) 1, -2, 4, …

i) ……………………………………………………………………………………

ii) ……………………………………………………………………………………

d) 1000, 100, 10, …

i) ……………………………………………………………………………………

ii) ……………………………………………………………………………………

e) -4, 16, -64, …

i) ……………………………………………………………………………………

ii) ……………………………………………………………………………………

f) 2, 20, 200, …

i) ……………………………………………………………………………………

ii) ……………………………………………………………………………………

g) -3, 6, -12, …

i) ……………………………………………………………………………………

ii) ……………………………………………………………………………………

h) 5, -25, 125, …

i) ……………………………………………………………………………………

ii) ……………………………………………………………………………………

6. Write down the first five terms of any sequence that is both arithmetic and geometric. Explain why the sequence is both arithmetic and geometric.

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*Level 5 – 6*

7. On week 1 Paul saved $10. In each subsequent week he saves 10% more than the previous week.

a) Determine an expression $t_n$ for the amount he saves on the $n^{\text{th}}$ week.

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b) Determine the first week when he saves at least $30.

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8. Determine the value(s) of the missing term in the following geometric sequences. Show your working out.

a) $t_1 = 5$, $t_4 = 40$, $t_3 = ?$          .................................................................

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b) $t_5 = -1$, $t_8 = 27$, $t_7 = ?$          .................................................................

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c) $t_{12} = 100$, $t_{16} = 1 \times 10^{-2}$, $t_{10} = ?$   ................................................................

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d) $t_8 = 2$, $t_{13} = -64$, $t_5 = ?$   ................................................................

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9. Determine the value(s) of $k$ in the following geometric sequences.

a) 2, $3k - 1$, 32

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b) $k$, $k + 1$, $k + 3$

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c) $k^2$, $k^2 + 2$, $k^2 + 8$

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*Level 7 – 8*

10. The $1^{st}$, $4^{th}$ and $13^{th}$ terms of an arithmetic sequence form the first three terms of a geometric sequence. If the first term of the arithmetic sequence is 3, find the common difference, $d$.

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11. The sum of the first two terms of a geometric sequence is 84. The sum of the first three terms is 196. Find the value(s) of $a$ and $r$. You should get two different sets of answers.

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12. Consider $S_n$ the sum of the first $n$ terms of a geometric sequence:

$$S_n = a + ar + ar^2 + \cdots + ar^{n-1}$$

a) Show that the value of $S_n$ is equal to $\dfrac{a(1-r^n)}{1-r}$.

*Hint: Multiply both sides by $(1-r)$ and expand and simplify the right side*

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b) Use the formula you have just derived to evaluate the following:

i) $4000 + 2000 + 1000 + \cdots + 62.5$

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ii) $1 + 3 + 9 + \cdots + 6561$

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iii) $(-3) + 6 + (-12) + 1536$

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