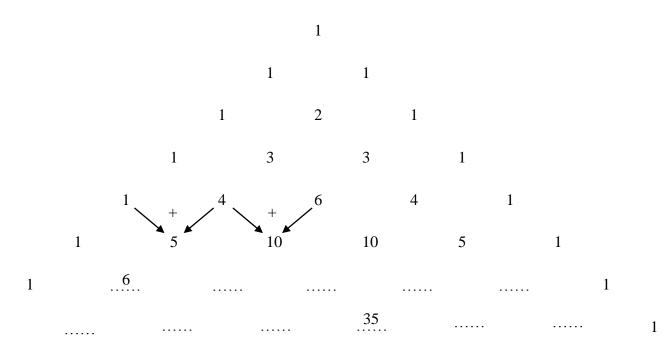
## **Binomial Expansion**

## *Level 1 – 2*

1

To determine a number in Pascal's triangle we simply calculate the sum of the two numbers above.

1. Complete the missing values:



2. We can use a function to express any number in Pascal's triangle. The  $r^{th}$  number in the  $n^{th}$  row is written

$${}^{n}C_{r}$$
 or  $\binom{n}{r}$ 

Note that we start counting both n and r from zero. For example:

$$\binom{4}{2} = 6$$
  $\binom{5}{4} = 5$   $\binom{3}{0} = 1$   $\binom{3}{0} = 1$ 

a) Use the previous question to write down the following values:



We can use a calculator to determine any number in Pascal's triangle.

Example: Use a TI-84 to determine  ${}^{8}C_{3}$ .





and select nCr from the PRB menu and then press





All other scientific calculators also have an nCr function.

Use your calculator to determine:

a) <sup>10</sup>C<sub>2</sub> .....

b)  $\binom{12}{5}$  .....

- $c) \begin{pmatrix} 15 \\ 3 \end{pmatrix} \qquad \dots$

d)  ${}^9C_7$  .....

*Level 3 – 4* 

3. There is also an equation we can use to calculate the value of  ${}^{n}C_{r}$ :

$${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

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 where  $n! = n \times (n-1) \times (n-2) \times ... \times 2 \times 1$ 

Example

$$^{7}C_{2} = \frac{7!}{2!(7-2)!}$$

$$=\frac{\overbrace{7\times 6\times 5\times 4\times 3\times 2\times 1}^{7!}}{\underbrace{2\times 1}_{2!}\times \underbrace{5\times 4\times 3\times 2\times 1}_{(7-2)!=5!}}$$

$$=\frac{7\times6\times5\times4\times3\times2\times1}{2\times1\times5\times4\times3\times2\times1}$$

$$=\frac{42}{2}=21$$

By hand, determine the following. Show full working out:

a)  ${}^{6}C_{4}$ 

	b) $\binom{8}{2}$				
	c) <sup>9</sup> C <sub>7</sub>				
4.	Expand and simplify the following. The first two have been completed. The third has been <i>started</i> <b>Show full working out</b> .				
	a) $(x+1)^2$	$(x+1)(x+1) = x^2 + x + x + 1$			
		$=x^2+2x+1$			
	b) $(x+1)^3$	$(x+1)(x+1)^2 = (x+1)(x^2+2x+1)$			
		$= x^3 + x^2 + 2x^2 + 2x + x + 1 = x^3 + 3x^2 + 3x + 1$			
	c) $(x+1)^4$	$(x+1)(x+1)^3 =$			
	d) $(x+1)^5$				

5.	Look at your answers to the previous question and describe any patterns you see. <i>Hint: compare your answers to the numbers in Pascal's triangle.</i>				
5.	Determine the coefficient of $x^3$ ( = the number in front of $x^3$ ) in the expansion of $(x+1)^7$ .				
7.	Determine the coefficient of $x^6$ in the expansion of $(x+1)^9$ .				
8.	Determine the coefficient of $x^{10}$ in the expansion of $(x+1)^{16}$ .				
9.	Determine the coefficient of $x^3$ in the expansion of $(x+1)^{20}$ .				
10.	Determine the coefficient of $x^5$ in the expansion of $(x+1)^{22}$ .				
_ 0.	=				

11. Expand and simplify the following:					
a) $(a+b)^2$					
b) $(a+b)^3$					
c) $(a+b)^4$					
d) $(a+b)^5$					
12. Use your results or any patterns you notice in the previous question to determine the coefficient of					
a) $x^3$ in the expansion	$1 \circ f(2x+3)^5$				
b) $x$ in the expansion of $(2-x)^4$					
c) $x^5$ in the expansion	$1 \operatorname{of} (-2x+5)^7$				