Arithmetic Series Introduction

(Criterion B Formative Assessment)

An arithmetic series is the sum of an arithmetic sequence. For example:

$2+6+10+14+18$

$1+2+3+...+10$ (This means $1+2+3+4+5+6+7+8+9+10$)

$7+10+13+...+40$ (This means $7+10+13+16+19+22+25+28+31+34+37+40$)

Level 1 – 2

1. Write down the value of the following series:

   a) $1+2+3+4$ ……

   b) $2+5+8+11+14$ ……

   c) $10+6+2+(-2)$ ……

   d) $8+13+18+...+33$ ……

   e) $10+12+14+...+30$ ……

   f) $(-3)+(-5)+(-7)+...+(-19)$ ……

   g) $2+(-1)+(-4)+...+(-16)$ ……

2. In each of the following, write down how many terms are being added together. For example, in the series $2+5+8+11+14+17$ there are 6 terms being added together. In the series $3+5+7+9$ there are 4 terms being added together.

   a) $9+7+5+3+1$ ……

   b) $15+18+21+24+27+30+33$ ……

   c) $(-4)+(-7)+(-10)+(-13)$ ……

   d) $7+11+15+...+51$ ……

   e) $3+10+17+...+45$ ……

   f) $1+7+13+...+37$ ……

   g) $3+(-1)+(-5)+...+(-41)$ ……

3. If the first term is t_1 and the last term is t_n show that the number of terms in a series is equal to

$$n = \frac{t_n - t_1}{d} + 1.$$

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4. Consider the following series:

$$4 + 7 + 10 + 13 + 16 + 19 + 22 + 25$$

a) Write down the value of the following:

i) $t_1 + t_8$ …… ii) $t_2 + t_7$ ……

iii) $t_3 + t_6$ …… iv) $t_4 + t_5$ ……

b) What do you notice about your answers? ……

c) Write down a simple multiplication which represents the value of this series. Calculate this value.

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5. Consider the following series:

$$(-2) + 2 + 6 + 10 + 14 + 18 + 22 + 26 + 30 + 34 + 38 + 42$$

a) Write down the value of the following:

i) $t_1 + t_{12}$ …… ii) $t_2 + t_{11}$ ……

iii) $t_3 + t_{10}$ …… iv) $t_4 + t_9$ ……

v) $t_5 + t_8$ …… vi) $t_6 + t_7$ ……

b) What do you notice about your answers? ……

c) Write down a simple multiplication which represents the value of this series. Calculate this value.

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Level 5 – 6

6. For the following series

i) Write down the common difference
ii) Determine how many terms there are
iii) Find the sum of the first and last term
iv) Write down a simple multiplication which represents the value of this series. Calculate this value.

a) $1+2+3+...+10$

i) ……

ii) ……

iii) ……

iv) ……

b) $3+9+15+...+45$

i) ……

ii) ……

iii) ……

iv) ……

c) $4+11+18+...+1411$

i) ……

ii) ……

iii) ……

iv) ……

d) $(-9)+(-13)+(-17)+...+(-213)$

i) ……

ii) ……

iii) ……

iv) ……

Level 7 – 8

7. Consider the series $4+8+12+...+404$.

   a) Complete the following:

	S	=	4	+	8	+	12	+	…	+	400	+	404
+	S	=	404	+	400	+	____	+	…	+	___	+	___
	$2S$	=	___	+	___	+	408	+	…	+	408	+	408

   b) Calculate how many terms there are. ……

   c) Write down an equation involving only *S*. Solve the equation to find the value of *S*.

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8. Using the same method as the previous question show that the sum S_n of the first *n* terms of an arithmetic sequence with common difference *d* is

$$S_n = \frac{n}{2}\left[2t_1 + (n-1)d\right]$$

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9. Show that an alternative formula for the sum S_n of the first n terms of an arithmetic sequence is

$$S_n = \frac{n}{2}(t_1 + t_n)$$

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10. Calculate

a) The sum of all odd numbers between 200 and 350.

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b) The sum of the first 200 even numbers greater than 10.

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c) The sum of the 50 largest odd numbers less than 300.

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d) The sum of the 80 smallest positive integers not divisible by 7.

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