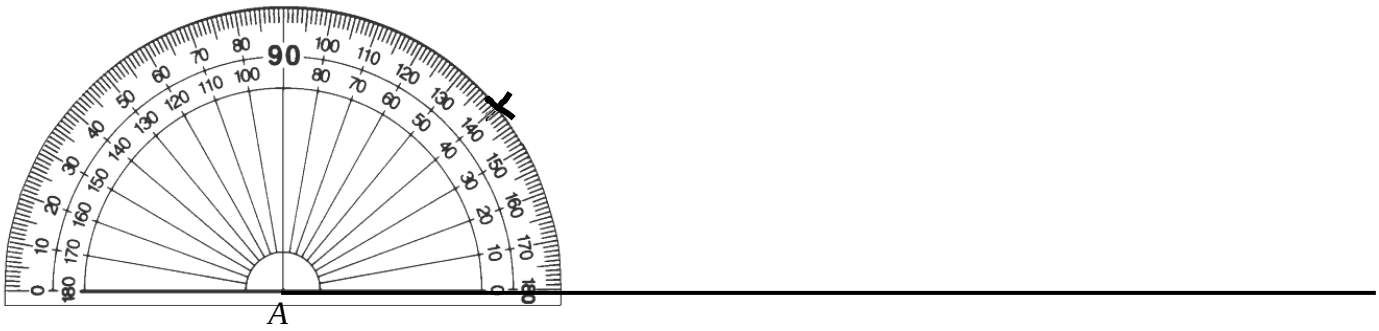
The Ambiguous Case of the Sine Rule Quiz

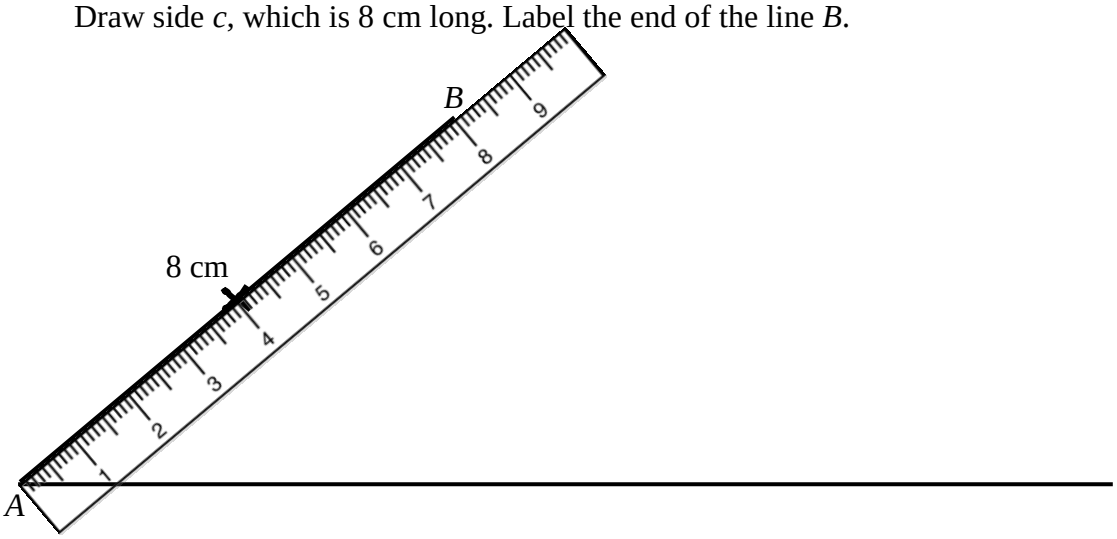
Example

Construct a triangle with $A = 40°$, $a = 6\,\text{cm}$ and $c = 8\,\text{cm}$.

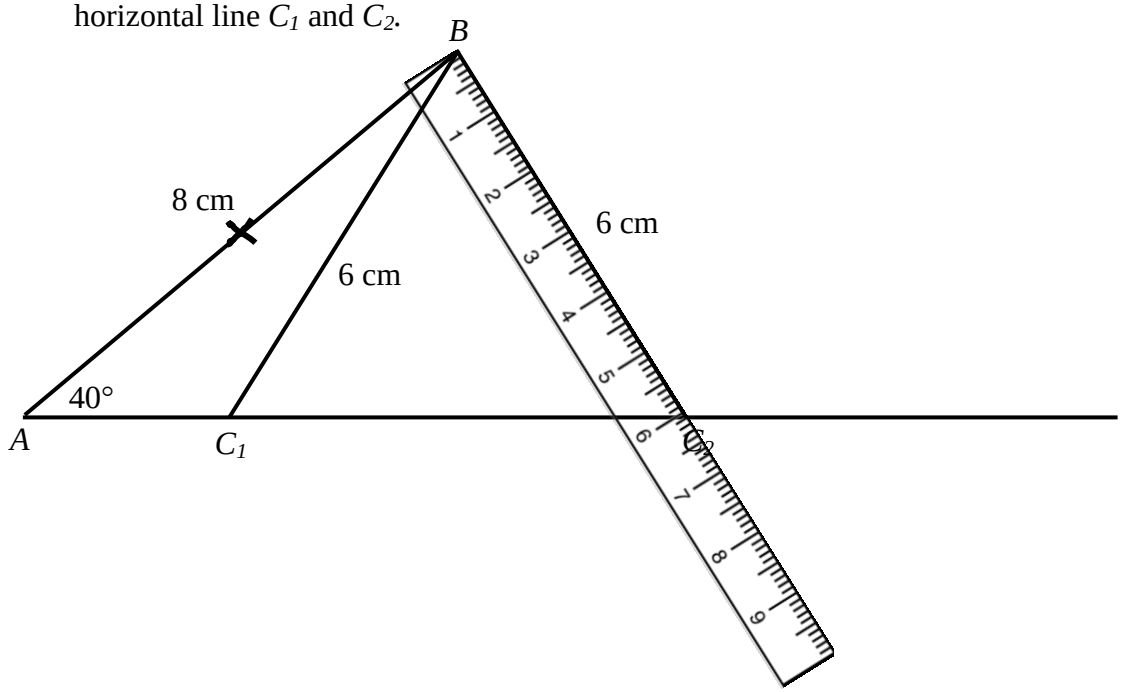
Step 1: Draw a long horizontal line. Label one end of the line *A*. Use a protractor to measure angle *A*.



Step 2: Draw side *c*, which is 8 cm long. Label the end of the line *B*.

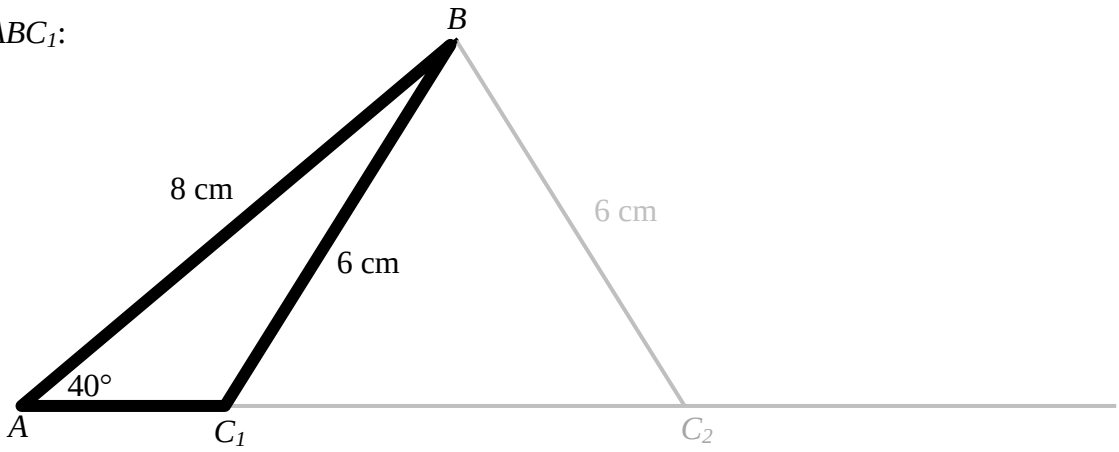


Step 3: Draw a line from B which is 6 cm long and meets the horizontal line. You should find that it is possible to draw two lines. Draw them both. Label the points where they meet the horizontal line C_1 and C_2.

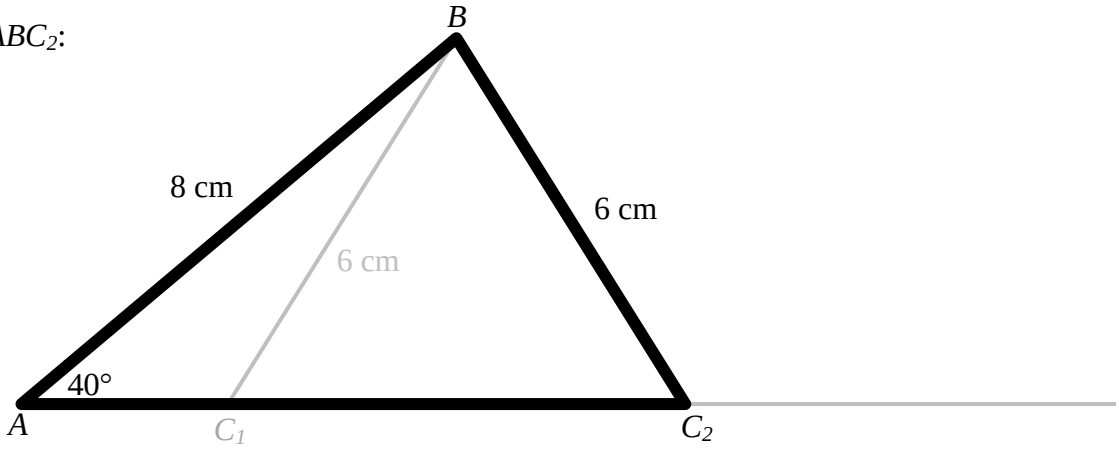


We have constructed two triangles which satisfy $A = 40°$, $a = 6\,\text{cm}$ and $c = 8\,\text{cm}$.

Triangle 1: ABC_1:



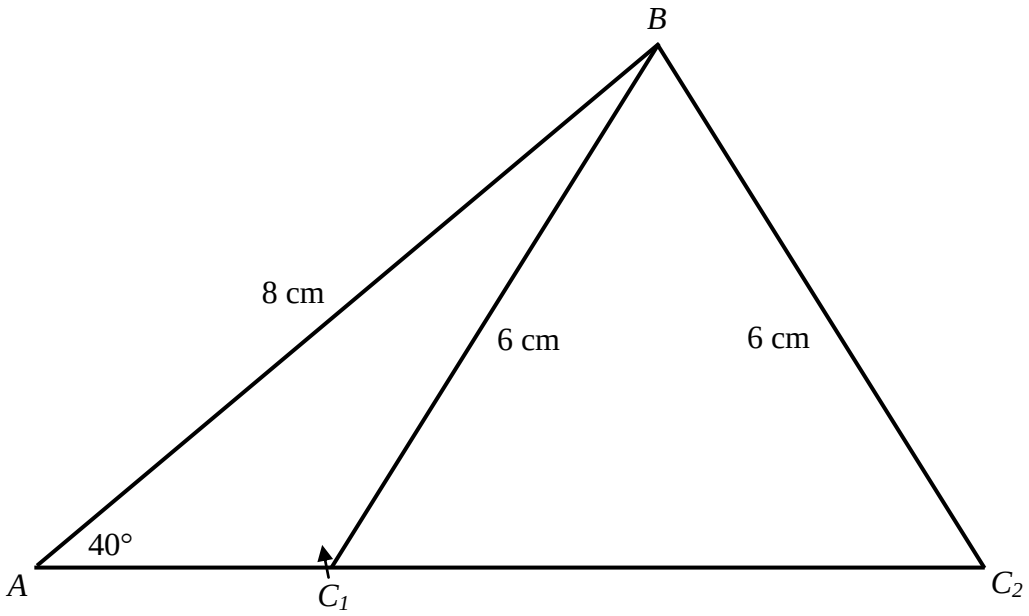
Triangle 2: ABC_2:



Complete the tasks on the following pages. Task 1 refers to this example…

Task 1: *(This corresponds to levels 1 – 2 of criterion A)*

- Use the sine rule to show that one value for angle C is $58.987°$. Have you calculated C_1 or C_2? Add this angle to the correct part of the diagram below.
- Use angle properties to calculate the size of every other angle in the diagram.
- Calculate the lengths of AC_1 and AC_2 to 2 decimal places.



Working Out

Summarize your results in the table below. Triangle 1 should be the smaller triangle:

Triangle	*A*	*B*	*C*	*a*	*b*	*c*
1	40°			6 cm		8 cm
2	40°			6 cm		8 cm

Task 2: *(This corresponds to levels 3 – 4 of criterion A)*

Find the lengths of all sides and the size of all angles in a triangle with $A = 29°$, $a = 5\,\text{cm}$ and $c = 9\,\text{cm}$.

Summarize your results in the table below. Triangle 1 should be the smaller triangle:

Triangle	*A*	*B*	*C*	*a*	*b*	*c*
1						
2						

Task 3: *(This corresponds to levels 5 – 6 of criterion A)*

Using only the sine rule determine how many triangles can be constructed from the following information. Use the empty spaces for any **unit circle** diagrams you need to draw.

a) $C = 54°,\ a = 10\,\text{cm},\ c = 9\,\text{cm}$

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b) $A = 30°,\ a = 6.5\,\text{cm},\ b = 13\,\text{cm}$

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c) $B = 37°,\ b = 14\,\text{cm},\ c = 14\,\text{cm}$

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d) $C = 23°,\ b = 14\,\text{cm},\ c = 15\,\text{cm}$

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Task 4: *(This corresponds to levels 7 – 8 of criterion A)*

If we use the cosine rule to determine missing lengths and it gives us two different positive lengths then it is possible to construct two different triangles using the given information.

a) Suppose we are given the size of angle *A*, the length of side *a* and the length of side *b*.

i) If the cosine rule gives us two different values for *c* show that $\cos^2 A > \frac{b^2 - a^2}{b^2}$

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ii) If the cosine rule gives us two different **positive** values for *c* also show that $b > a$

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b) **Hence** show that it is possible to construct two triangles if

$$b\sin A < a < b$$

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Formula Sheet

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\sin^2 A + \cos^2 A = 1$$

If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$