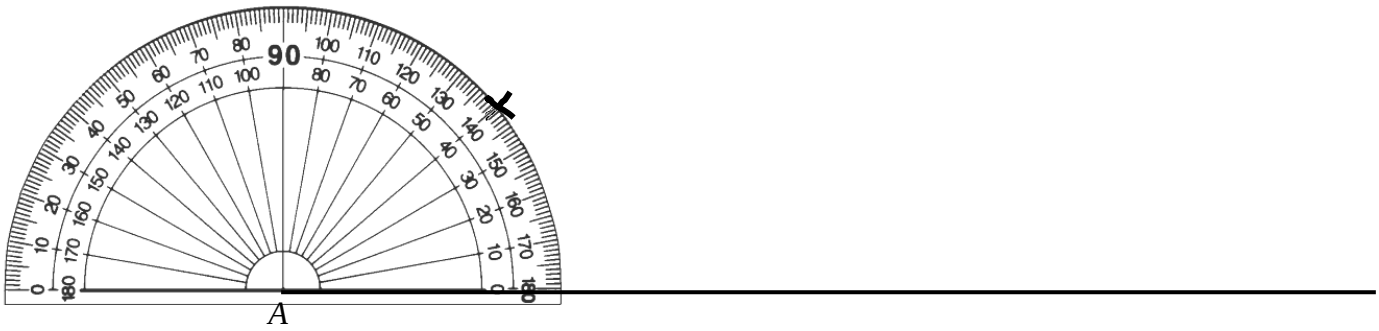
The Sine Rule – The Ambiguous Case

Criterion B Formative Investigation

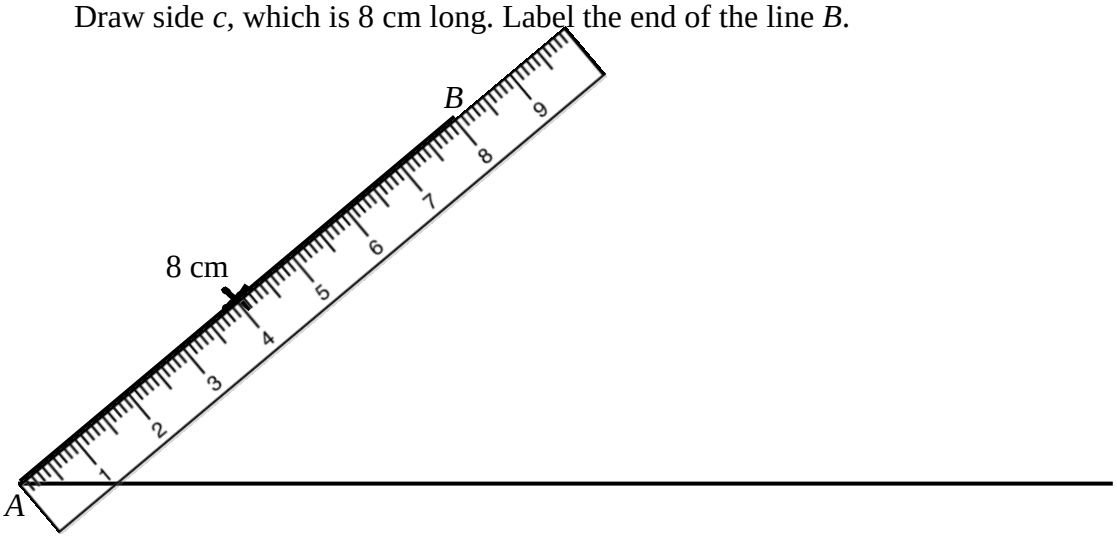
Example

Construct a triangle with $A = 40°$, $a = 6\,\text{cm}$ and $c = 8\,\text{cm}$.

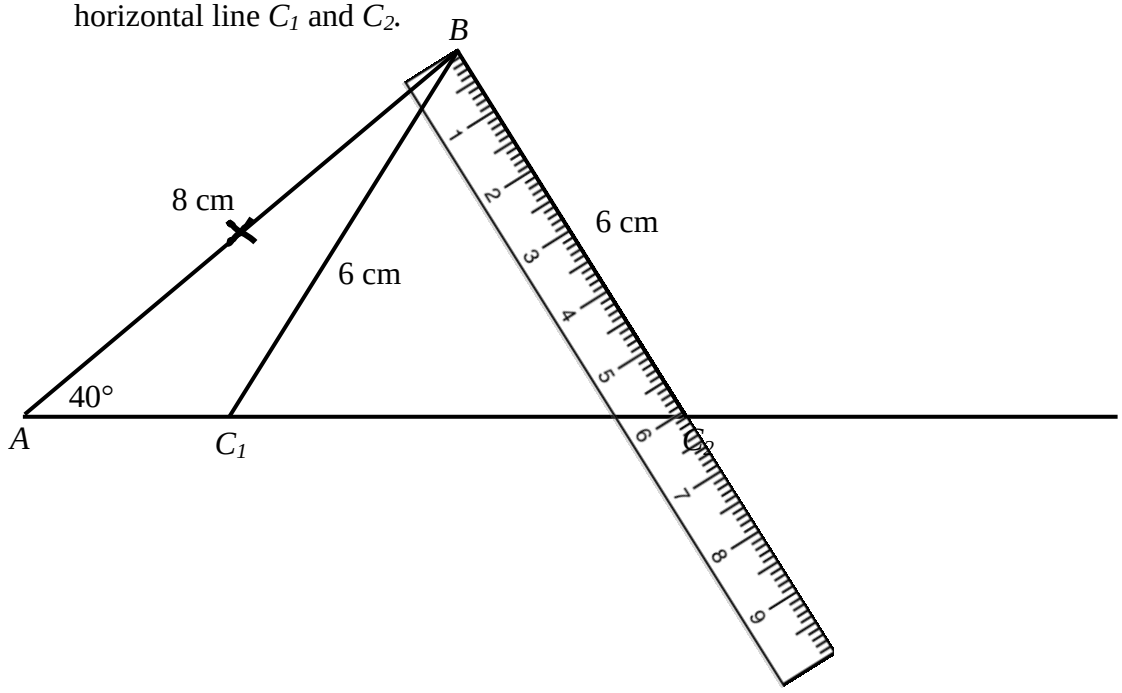
Step 1: Draw a long horizontal line. Label one end of the line *A*. Use a protractor to measure angle *A*.



Step 2: Draw side *c*, which is 8 cm long. Label the end of the line *B*.

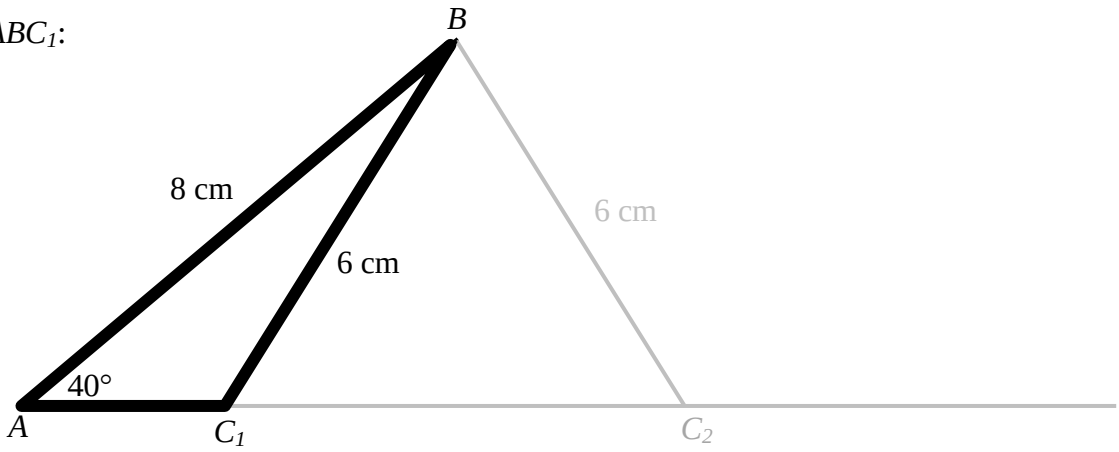


Step 3: Draw a line from B which is 6 cm long and meets the horizontal line. You should find that it is possible to draw two lines. Draw them both. Label the points where they meet the horizontal line C_1 and C_2.

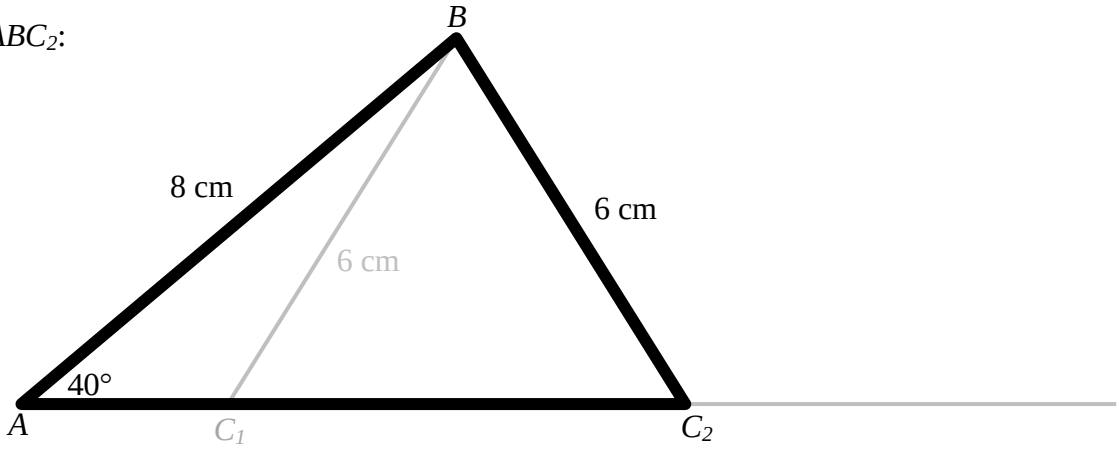


We have constructed two triangles which satisfy $A = 40°$, $a = 6\,\text{cm}$ and $c = 8\,\text{cm}$.

Triangle 1: ABC_1:



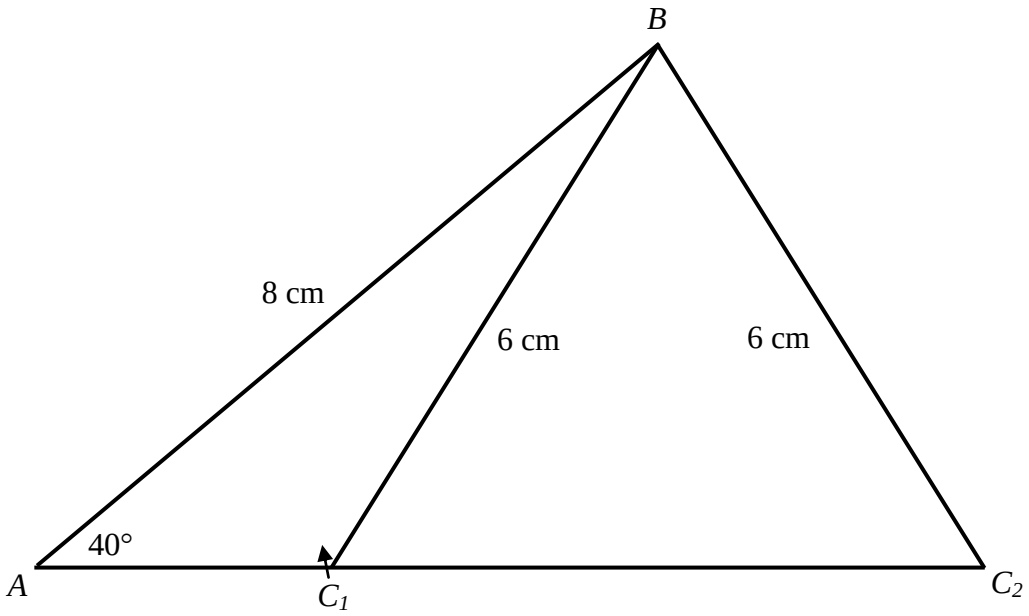
Triangle 2: ABC_2:



Complete the tasks on the following pages. Task 1 refers to this example…

Task 1: *(This corresponds to levels 1 – 2 of criterion B)*

- Use the sine rule to show that one value for angle C is $58.987°$. Have you calculated C_1 or C_2? Add this angle to the correct part of the diagram below.
- Use angle properties to calculate the size of every other angle in the diagram.
- Calculate the lengths of AC_1 and AC_2 to 2 decimal places.



Working Out

Summarize your results in the table below. Triangle 1 should be the smaller triangle:

Triangle	*A*	*B*	*C*	*a*	*b*	*c*
1	40°			6 cm		8 cm
2	40°			6 cm		8 cm

Task 2: *(This corresponds to levels 3 – 4 of criterion B)*

Find the lengths of all sides and the size of all angles in a triangle with $A = 45°$, $a = 8\,\text{cm}$ and $c = 10\,\text{cm}$.

Summarize your results in the table below. Triangle 1 should be the smaller triangle:

Triangle	*A*	*B*	*C*	*a*	*b*	*c*
1	45°			8 cm		10 cm
2	45°			8 cm		10 cm

Task 3: *(This corresponds to levels 3 – 4 of criterion B)*

Find the lengths of all sides and the size of all angles in a triangle with $C = 30°$, $c = 7$ cm and $b = 12$ cm.

Summarize your results in the table below. Triangle 1 should be the smaller triangle:

Triangle	*A*	*B*	*C*	*a*	*b*	*c*
1			30°		12 cm	7 cm
2			30°		12 cm	7 cm

Task 4: *(This corresponds to levels 5 – 6 of criterion B)*

By drawing accurate *scale* diagrams, determine how many triangles can be constructed using the given information (circle your choice).

a) $A = 35°$, $a = 6\,\text{cm}$ and $c = 9\,\text{cm}$

Number of triangles: 0 / 1 / 2

b) $B = 42°$, $b = 12\,\text{cm}$ and $c = 7\,\text{cm}$

Number of triangles: 0 / 1 / 2

c) $C = 50°$, $c = 55\,\text{cm}$ and $a = 90\,\text{cm}$

Number of triangles: 0 / 1 / 2

d) $A = 30°$, $a = 50\,\text{cm}$ and $c = 100\,\text{cm}$

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Number of triangles: 0 / 1 / 2

Task 5: *(This corresponds to levels 7 – 8 of criterion B)*

If we are given the size of angle A (which is less than $90°$) and the lengths of sides a and b, determine the relationship between A, a and b if we be able to construct i) one triangle ii) two triangles and iii) no triangles

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Use your rules to complete the following table:

Given Measurements	Number of possible triangles (0 / 1 / 2)	Reason
$A = 70°$, $a = 5\,\text{cm}$ and $c = 10\,\text{cm}$		
$B = 20°$, $b = 30\,\text{cm}$ and $c = 20\,\text{cm}$		
$A = 45°$, $a = 10\,\text{cm}$ and $c = 14\,\text{cm}$		
$C = 30°$, $c = 8\,\text{cm}$ and $a = 16\,\text{cm}$		

If you forgot your protractor, cut out one of these and use it: