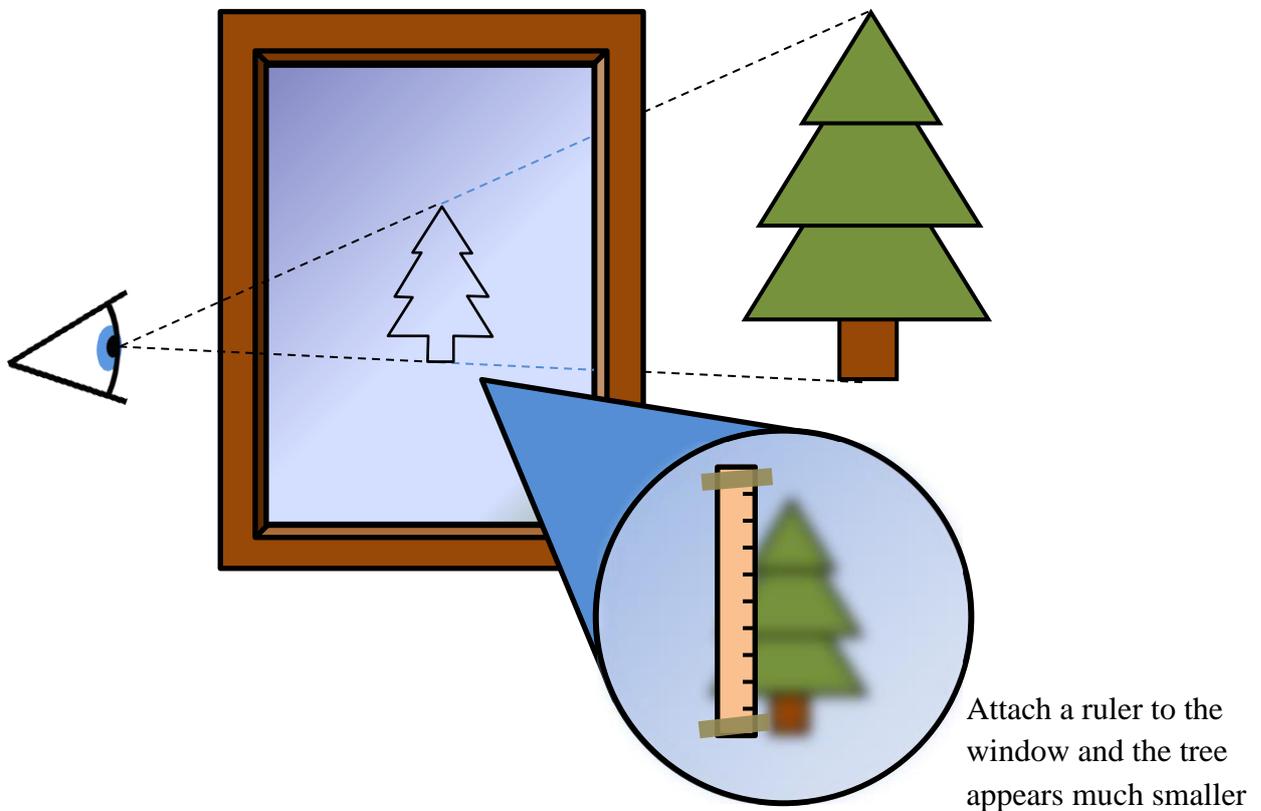# Why do we See Things Differently from Different Positions?

A person looks out of the window at a tree. If a ruler were attached to the window the tree would appear much smaller than its actual size.

Attach a ruler to the window and the tree appears much smaller

Task 1

Get any object and place it on the opposite side of a window from you. Measure the following:

- the distance d between you and the window
- the distance D between the object and the window
- the actual height H of the object
- the height of the object h as it appears in the window

Adjust only the value of D and take five different sets of measurements for approximately equally spaced values of D. Plot the values of D (x-axis) and h (y-axis) as a scatterplot.

Choose a different value of d and take five more sets of measurements using the same values of D as before. Add this data to your graph. Repeat this process one more time and add a third set of data to your graph.

Determine the general relationship between d, D, H and h. Justify your reasoning using explanations, calculations and diagrams. Use this relationship to add three curves to your graph (one for each of the three cases above). Use these curves to comment on the accuracy of your experiment.

Note that you may need help from your partner to take these measurements. You will also have to help your partner take his/her measurements, which should be based on a different object with a different size.

Consider a bus driving down the road. The windscreen of the bus is 1 metre in front of the driver and is completely vertical. The driver's eyes are 2 metres above the ground. There is a painted message on the road (a speed limit). When the bus is 5 metres away, the height of the message appears as though it is written on the windscreen with a height of 20 cm.
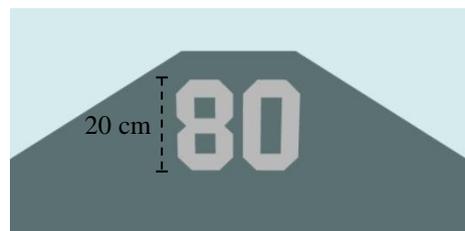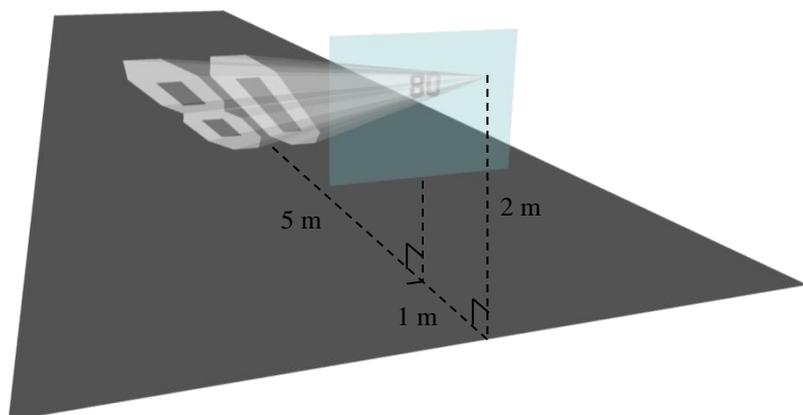

From the perspective of the driver, the message appears the same as if it were written on the windscreen with a height of 20 cm.

Task 2

Using the same conditions as above use Sketchup to design a message on the road that appears 20 cm tall in the windscreen. Before you start you should also decide the width you want the message to appear. Follow the Sketchup tutorial to create your message. You will have to determine the value of *x* in the diagram below and use this in your Sketchup model.
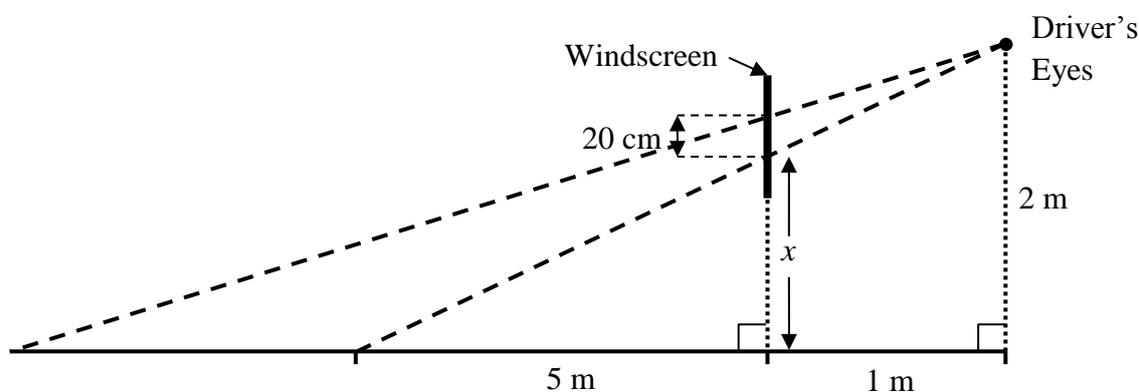
Now, suppose we don't have a tool such as Sketchup to help us. If we know the height (20 cm) and the width (your choice) we want the message to appear to have, how can we determine the size of *a*, *b* and *h* in the diagram below. Note that these letters represent lengths of sides of the bounding trapezoid. Justify your reasoning using explanations, calculations and diagrams. Use your solution to determine the accuracy of your Sketchup Model.
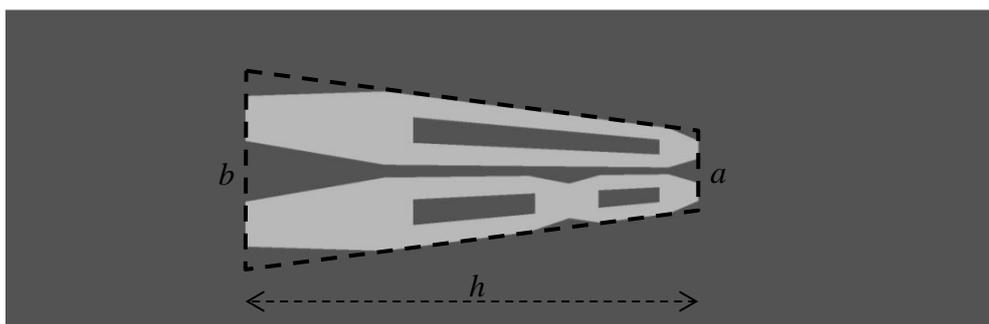
Submit your solutions to these tasks as a single report. The report should be able to stand on its own without having to refer to this document. Be sure to include:

- an introduction of what you are investigating for each task
- a brief explanation of how you will investigate it
- your solutions to tasks 1 and 2

All explanations should be in your own words, all diagrams should be created by yourself using a computer, all graphs should be created in Excel and all working out should be created using the equation editor. You may include a photograph to help clarify task 1. This should be a photograph of the experiment, and not a photograph of any working out or calculations done by hand.

Your report should NOT include:

- identical measurements to your partner's measurements for the experimental part of task 1
- an Excel tutorial
- a Sketchup tutorial (although a brief explanation of how you used Sketchup is expected)
- the same message as in the task 2 tutorial

If your report is not in Word format please submit it as a pdf file.

Criterion B: Investigating Patterns		
Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below.	
1 – 2	The student is able to: I. **apply, with teacher support**, mathematical problem-solving techniques to discover **simple patterns** II. **state predictions** consistent with patterns.	The student is able to: ○ perform an experiment that produces values of d, D, h and H. ○ attempt to determine the relationship between d, D, h and H.
3 – 4	The student is able to: I. **apply** mathematical problem-solving techniques to discover **simple patterns** II. **suggest general rules** consistent with **findings**.	The student is able to: ○ perform an experiment that produces reasonably accurate values of d, D, h and H. ○ attempt to determine the relationship between d, D, h and H. ○ attempt to determine how to calculate the dimensions of the painted message in task 2 without using Sketchup.
5 – 6	The student is able to: I. **select and apply** mathematical problem-solving techniques to discover **complex patterns** II. **describe patterns** as general rules consistent with **findings** III. **verify** the validity of these general rules.	The student is able to: ○ perform an experiment that produces reasonably accurate values of d, D, h and H. ○ determine the relationships between d, D, h and H in task 1 and add these relationships to the graph ○ determine how to calculate the dimensions of the painted message in task 2 without using Sketchup.
7 – 8	The student is able to: I. **select** and **apply** mathematical problem-solving techniques to discover **complex patterns** II. **describe patterns** as general rules consistent with **correct findings** III. **prove, or verify and justify**, these general rules.	The student is able to: ○ perform an experiment that produces reasonably accurate values of d, D, h and H. ○ determine the relationships between d, D, h and H in task 1 using clear explanations, calculations and diagrams and add these relationships to the graph. ○ determine how to calculate the dimensions of the painted message in task 2 using clear explanations, calculations and diagrams without using Sketchup.

Criterion C: Communication in Mathematics		
Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below.	
1 – 2	The student is able to: I. **use** limited mathematical language II. **use** limited forms of mathematical representation to present information III. **communicate** through lines of reasoning that are difficult to interpret.	The student is able to: o attempt to explain what is being investigated and how it is being investigated o create and format graph(s) using Excel
3 – 4	The student is able to: I. **use** some appropriate mathematical language II. **use** different forms of mathematical representation to present information adequately III. **communicate** through lines of reasoning that are able to be understood, although these are not always clear IV. adequately **organize** information using a logical structure.	The student is able to: o attempt to explain what is being investigated and how it is being investigated o use diagrams to justify explanations and calculations o create and format graph(s) using Excel o create a report that is able to be understood without referring to the task sheet
5 – 6	The student is able to: I. usually **use** appropriate mathematical language II. usually **use** different forms of mathematical representation to present information correctly III. move between different forms of mathematical representation with some success IV. **communicate** through lines of reasoning that are clear although not always coherent or complete V. present work that is usually **organized** using a logical structure.	The student is able to: o explain what is being investigated and how it is being investigated o use diagrams to justify explanations and calculations o move between explanations, calculations, tables and diagrams with appropriate linking sentences (the following table shows, figure 3 demonstrates, etc.) o export Sketchup models as 2D images o create and format clear and accurate graph(s) that satisfy all the requirements of task 1 o create a report that is able to be understood without referring to the task sheet
7 – 8	The student is able to: I. consistently use appropriate mathematical language II. use different forms of mathematical representation to consistently present information correctly III. move effectively between different forms of mathematical representation IV. communicate through lines of reasoning that are complete and coherent V. present work that is consistently organized using a logical structure.	The student is able to: o clearly explain what is being investigated and how it is being investigated o display formulae clearly and accurately using the equation editor o use clear and accurate diagrams to justify explanations and calculations o move effectively between explanations, calculations, tables and diagrams with appropriate linking sentences (the following table shows, figure 3 demonstrates, etc.) o export Sketchup models as 2D images o create and format one clear and accurate graph that satisfies all the requirements of task 1 o make good use of space on the page (no unnecessary white space, items positioned thoughtfully etc.) o create a report that is able to be understood without referring to the task sheet

| Criterion D: Applying Mathematics in Real Life Contexts		
Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below.	

1 – 2	The student is able to:
	  I. **identify** some of the elements of the authentic real-life situation
	  II. **apply** mathematical strategies to find a solution to the authentic real-life situation, with limited success.
		The student is able to:
		
		o attempt determine mathematical models to tasks 1 and 2
		o attempt to create a Sketchup model

3 – 4	The student is able to:
	  I. **identify** the relevant elements of the authentic real-life situation
	  II. **select**, with some success, adequate mathematical strategies to model the authentic real-life situation
	  III. **apply** mathematical strategies to reach a solution to the authentic real life situation
	  IV. **describe** whether the solution makes sense in the context of the authentic real-life situation.
		The student is able to:
		
		o attempt determine mathematical models to tasks 1 and 2
		o attempt to explain the accuracy of the experiment in task 1 by comparing it to the mathematical model
		o create a Sketchup model
		o attempt to explain the accuracy of the Sketchup model by comparing it to the mathematical model

5 – 6	TThe student is able to:
	  I. **identify** the relevant elements of the authentic real-life situation
	  II. **select** adequate mathematical strategies to model the authentic real-life situation
	  III. **apply** the selected mathematical strategies to reach a valid solution to the authentic real-life situation
	  IV. **describe** the degree of accuracy of the solution
	  V. **discuss** whether the solution makes sense in the context of the authentic real-life situation.
		The student is able to:
		
		o determine mathematical models to tasks 1 and 2
		o explain the accuracy of the experiment in task 1 by comparing it to the mathematical model
		o create an accurate Sketchup model and briefly explain how the model was created
		o explain the accuracy of the Sketchup model by comparing it to the mathematical model

7 – 8	The student is able to:
	  I. **identify** the relevant elements of the authentic real-life situation
	  II. **select** appropriate mathematical strategies to model the authentic real life situation
	  III. **apply** the selected mathematical strategies to reach a correct solution
	  IV. **explain** the degree of accuracy of the solution
	  V. explain whether the solution makes sense in the context of the **authentic** real-life situation.
		The student is able to:
		
		o determine accurate mathematical models to tasks 1 and 2
		o explain the accuracy of the experiment in task 1 by comparing it to the mathematical model
		o create an accurate Sketchup model and *briefly* explain, with screenshots, how the model was created
		o explain the accuracy of the Sketchup model by using appropriate Sketchup tools and comparing to the mathematical model

Excel Tutorial

Creating a Scatterplot

Suppose we wish to use Excel to create a scatterplot of the data in the table below.
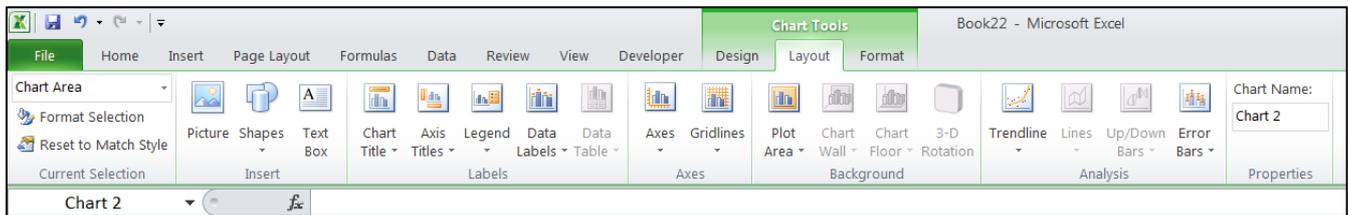
Enter the data into two columns in Excel. Highlight the two columns and click *Insert > Scatter > Scatter with only Markers*.

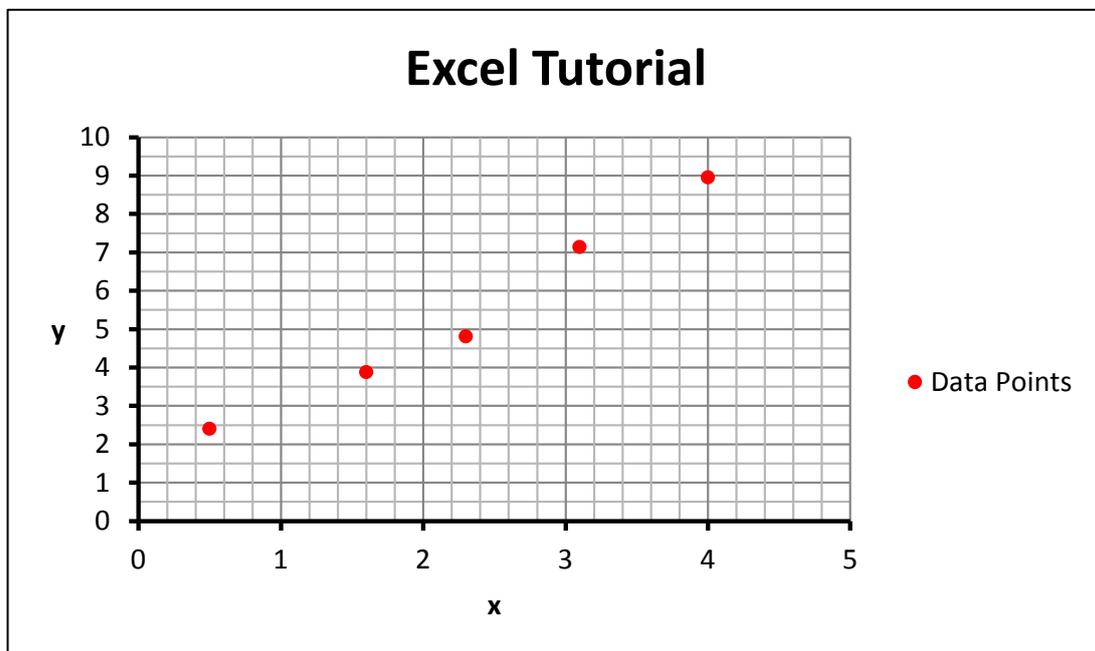| x	y
0.5	2.4
1.6	3.87
2.3	4.8
3.1	7.13
4	8.95
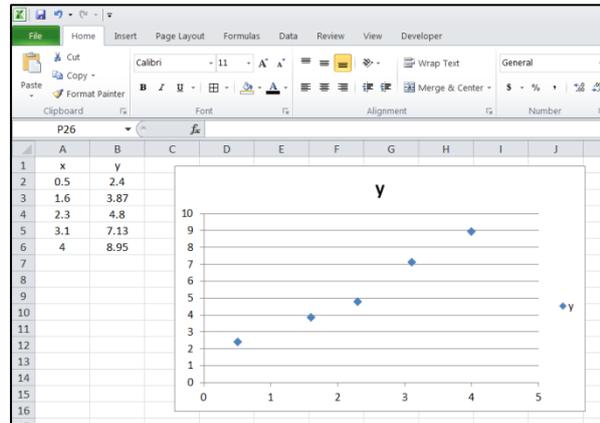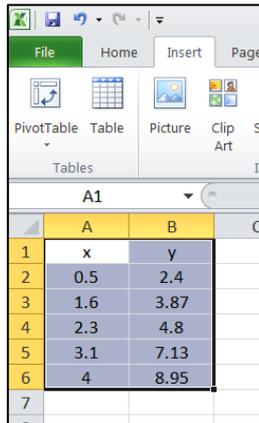
Formatting your Graph

Click on the graph and select various options from the *Layout* tab to make the graph more attractive.

Note that the content of cell B1 represents the name of the series. In the example below cell B1 has been renamed to "Data Points".

Adding New Data to the Graph

Suppose we wish to add the line $y = 2x + 1$ to the graph.

Enter the following into your spreadsheet. Note that when you press *Enter* after typing in an equation the value of the equation will be displayed in the cell, and not the equation itself.
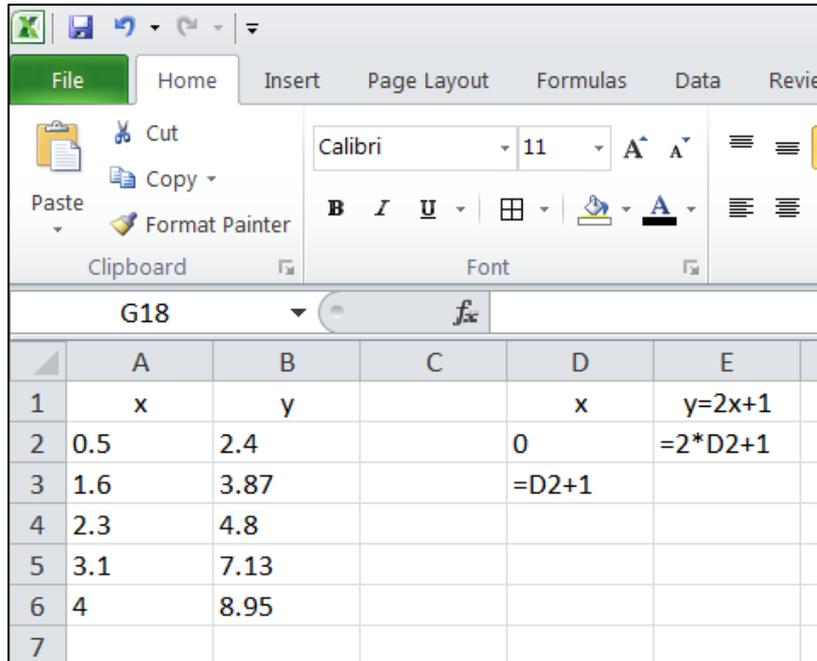
Copy and paste cell D3 to cells D4 to D6. Copy and paste cell E2 to cells E3 to E6. Your spreadsheet should now look like the following:
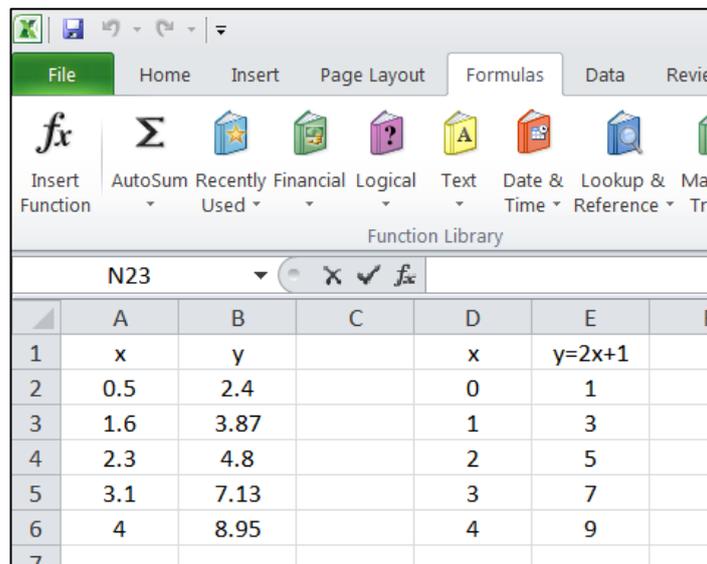
We now need to add this data to our graph.

Right-click on the graph and select *Select Data*. A screen similar to the one on the right should appear. Then click *Add*.
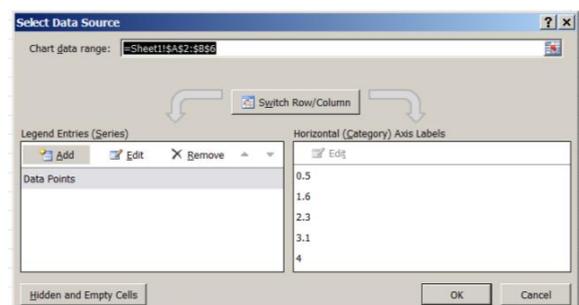
Place the cursor is in the *Series Name* textbox and then click on cell E1.

Place the cursor in the *Series X values* textbox and then highlight cells D2 to D6.

Place the cursor in the *Series Y values*, delete the current contents, and then highlight cells E2 to E6.

Your window should now look like the following.

Edit Series	? ×
Series name:	
=Sheet1!$E$1	= y=2x+1
Series X values:	
=Sheet1!$D$2:$D$6	= 0, 1, 2, 3, 4
Series Y values:	
=Sheet1!$E$2:$E$6	= 1, 3, 5, 7, 9
	OK Cancel
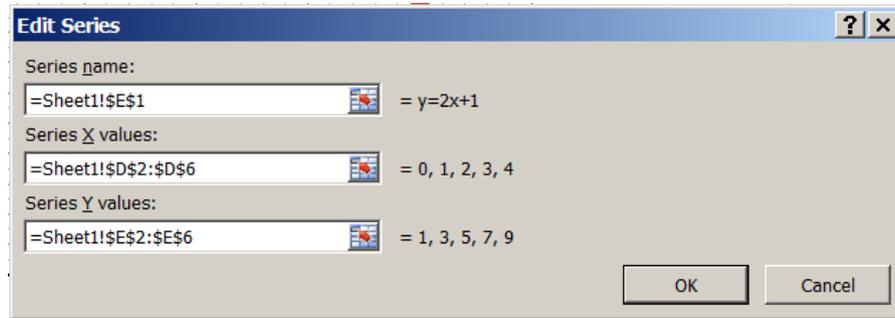
Click *OK* twice and the new data will appear on the graph. However, we want the new data to appear as a line, and not as a series of points. Right-click on any of the new data points and select *Change Series Chart Type*, then select *Scatter with Smooth Lines* and click *OK*.

Your graph should now look something like similar to this:

Excel Tutorial graph showing y=2x+1 line with Data Points
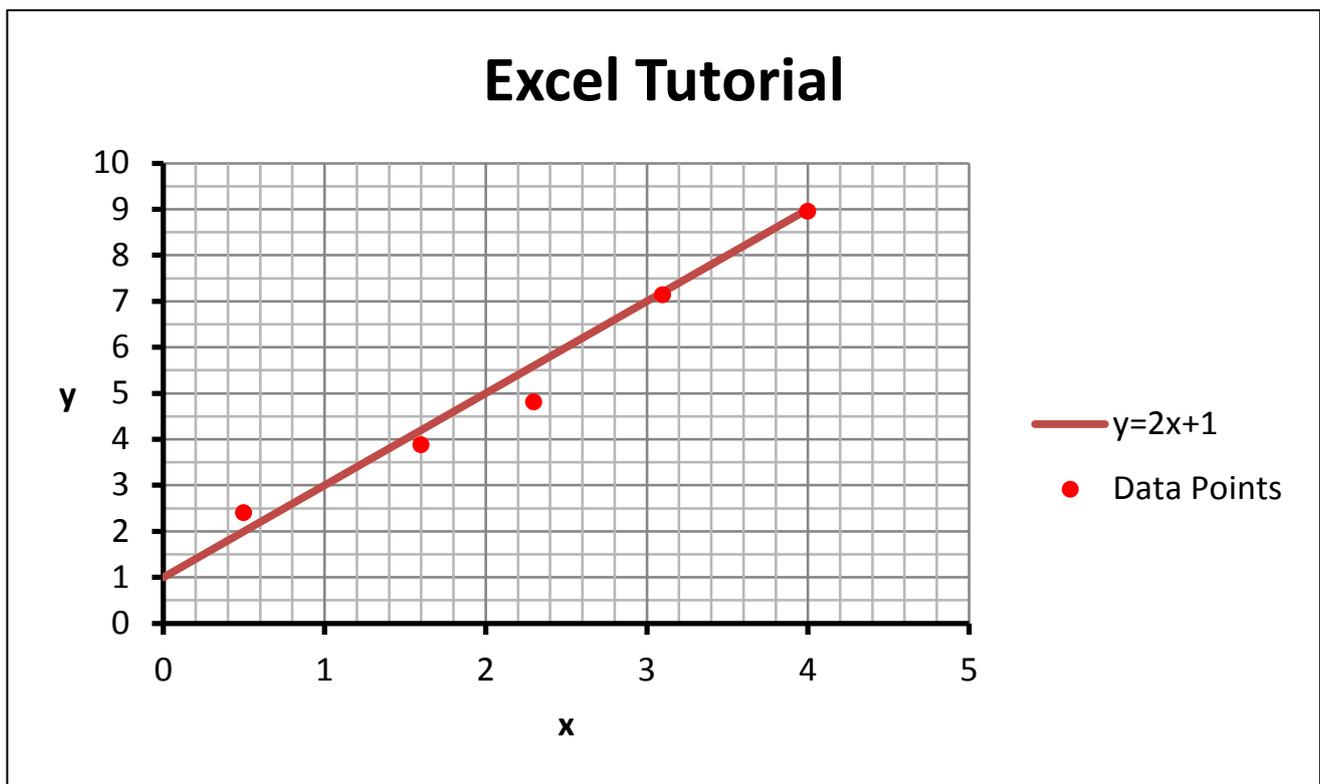
Sketchup Tutorial

Copy Arrays

Copy arrays enables us to quickly make multiple copies of an object which are each positioned a fixed distance apart.

For example, suppose we wish to make a grid containing many squares with sides of length 1 metre.
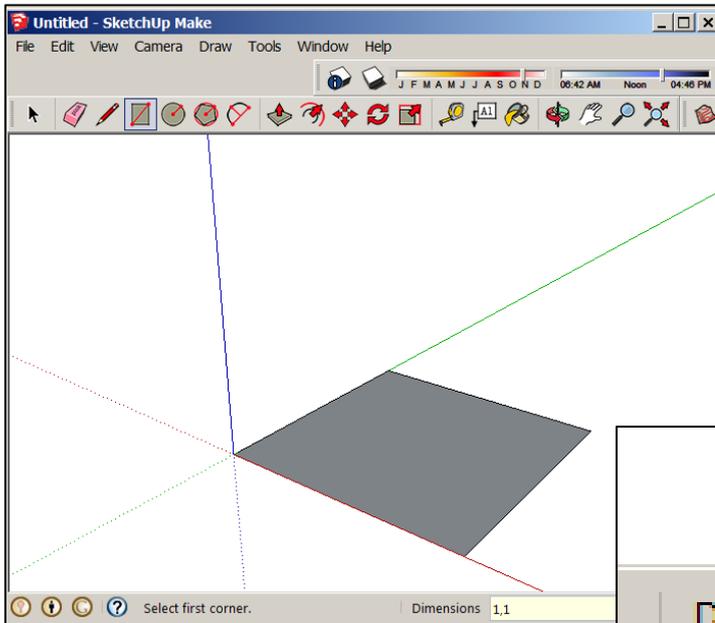

Draw a rectangle of any size and then immediately type 1,1 and press *Enter*. Note that you do not need to click inside the *Dimensions* text box before typing.

The rectangle will now be a square with sides of length 1 metre.
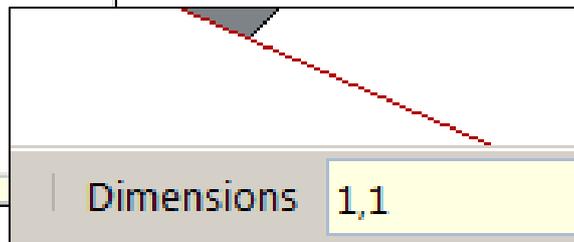
Select the square using the *Select* tool. Then select the *Move* tool. Hold down *Ctrl*, click once on the left vertex (=corner), then click once on the top vertex, then release *Ctrl*. Immediately type *x9* and press *Enter*. Nine copies of the square will be created, so there should be ten in total. This is called a *Copy Array*.
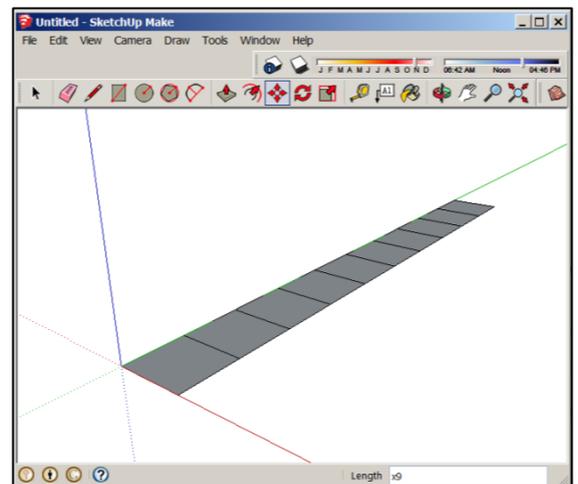
Now, select every square by triple clicking on any one of them. Perform one more copy array to create a grid containing 100 squares.

You may need to create a grid like this later on to help you create your message.

You do not need to save this grid. Once you are comfortable creating a copy array, delete it and move on to the next tutorial.
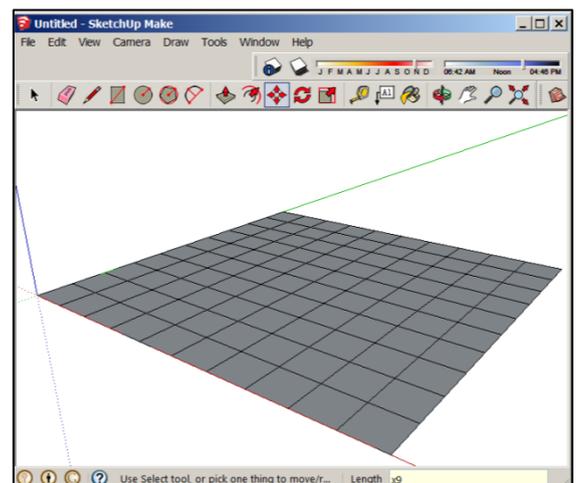
Creating the Message

Draw a rectangle with a width of 6 metres and a length of 50 metres. Recall how to specify the dimensions of a rectangle from the previous section.

This rectangle will represent the road. If we need to make it larger later on we can do so by selecting the *Move* tool and moving any edge of the rectangle to a new position.
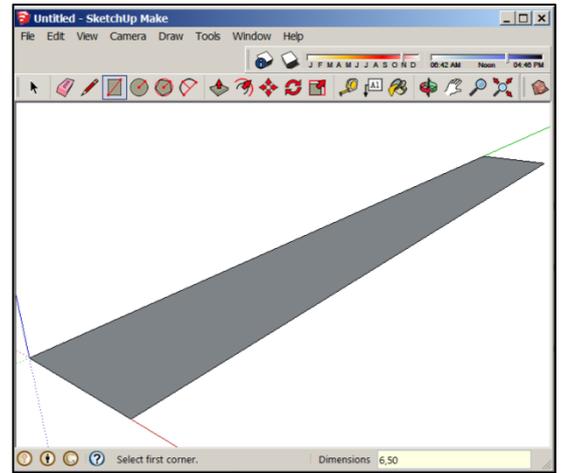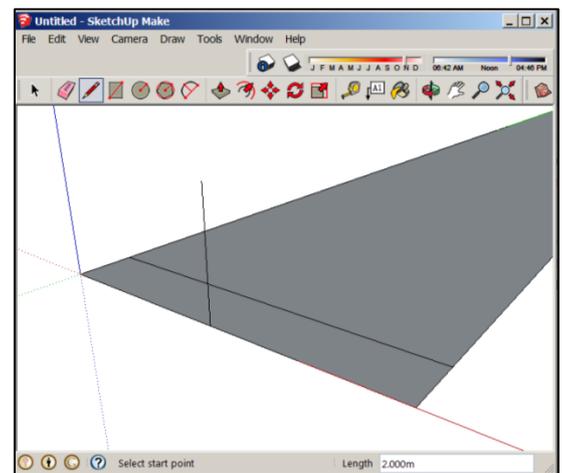
At one end of the rectangle draw another rectangle with a width of 6 m and a length of 1 m.
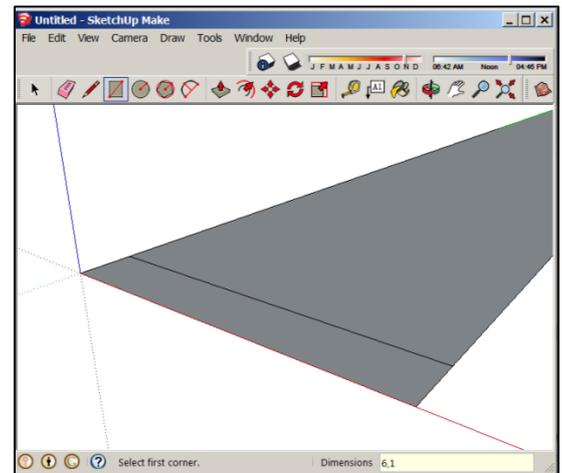
From the midpoint of the shortest side of the 6 m by 50 m rectangle draw a vertical line which is 2 m long. Note that *Sketchup* should automatically lock on to the midpoint of the line when the cursor is close to it.

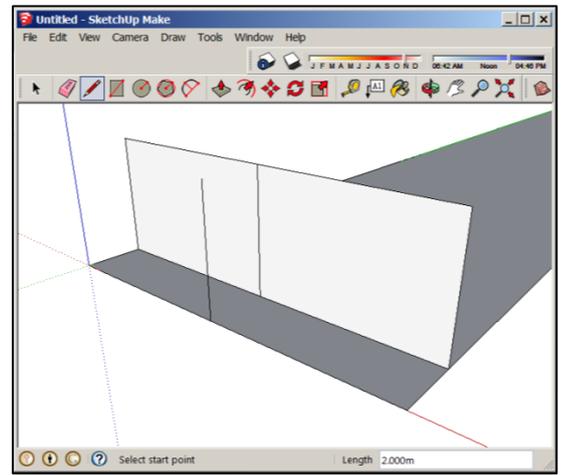The tip of this line will represent the position of the driver's eye.

From the long edge of the 6 m by 1 m rectangle draw a vertical rectangle with dimensions 6 m by 2 m.
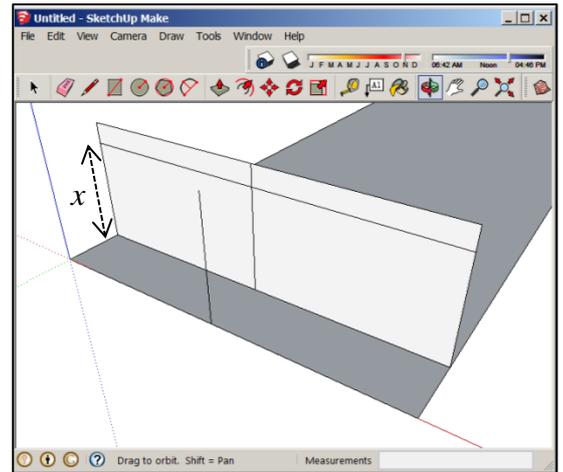
This will represent the windscreen of the bus.
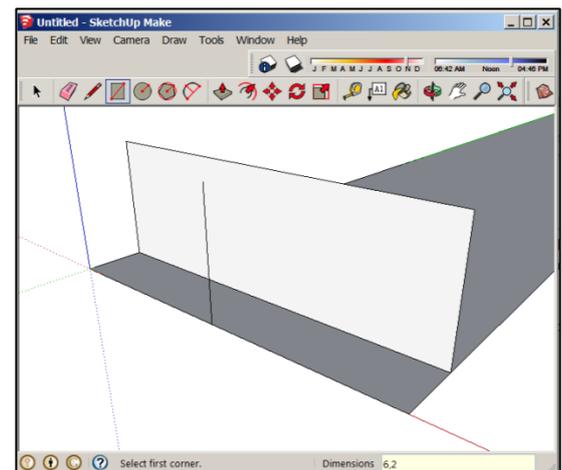
Draw a vertical line down the middle of the vertical rectangle. This will help us position our message.

Draw a vertical rectangle from the bottom left corner of the existing vertical rectangle. The height of this new rectangle should be equal to the value of x you calculated in the description of task 2.


We are now ready to create our message. If it helps, create a grid using a copy array and use the grid to help you create your message. **Make sure the message is centred, has a height of 20 cm, and a width equal to whatever you decided in the description of task 2. Do not use any curved lines in your message.**
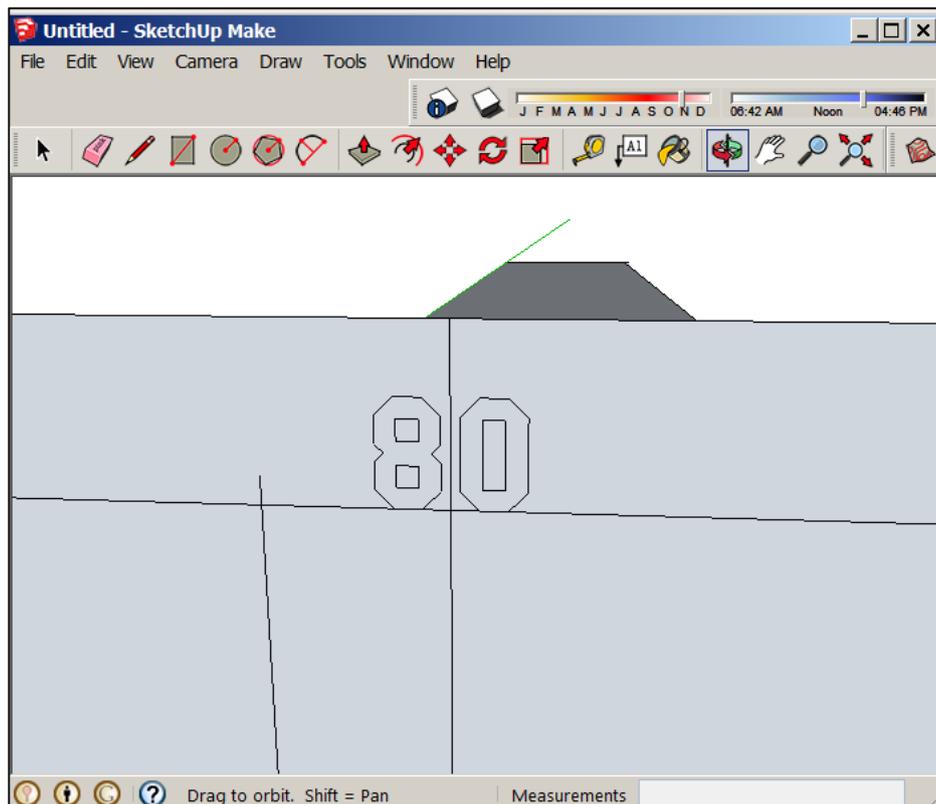
When you are satisfied with your message you can delete the vertical rectangle. All that you should have remaining is the road, the vertical line which represents the driver's eyes, and the message.
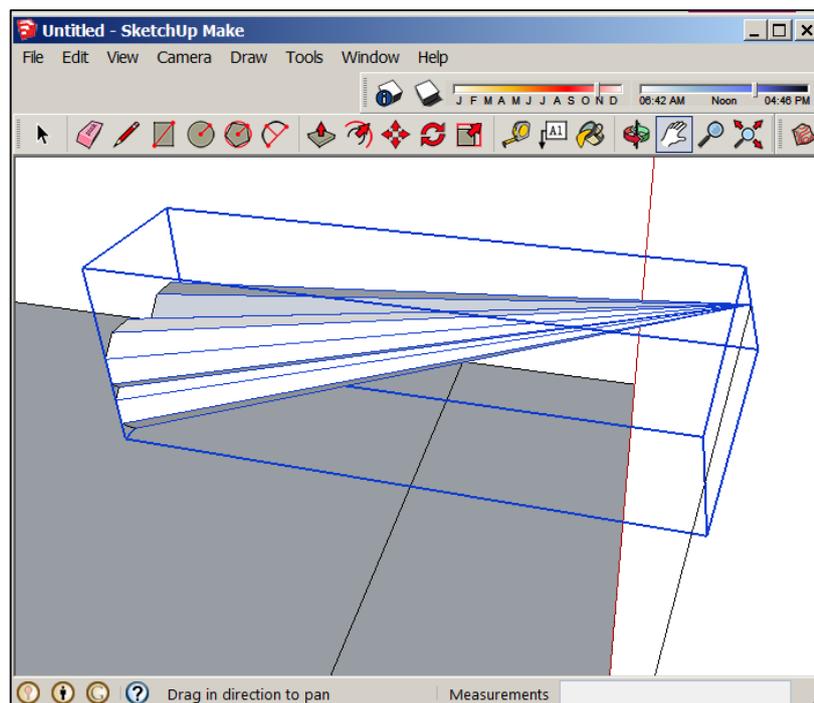
Now, select everything and group together. This is an important step. If you do not do this it will make it much more difficult later on.
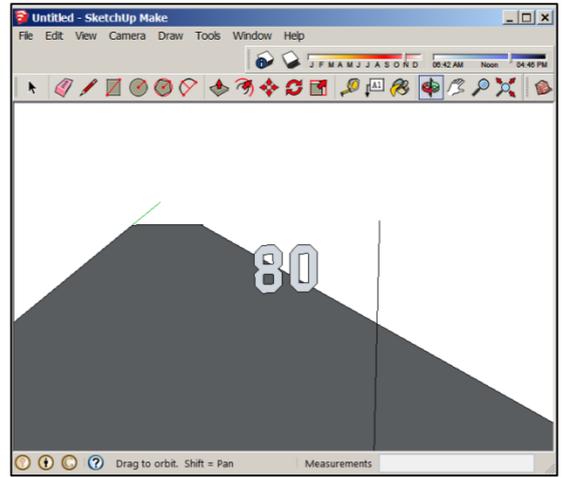
Draw triangles using the point representing the eyes and adjacent vertices of the message. Connect inside vertices first.
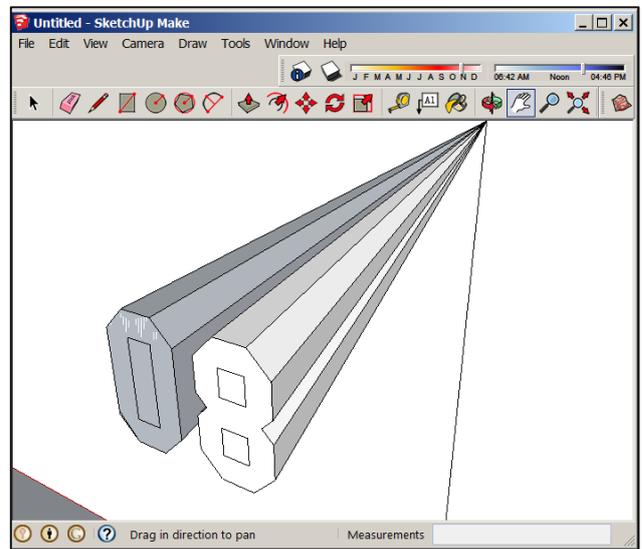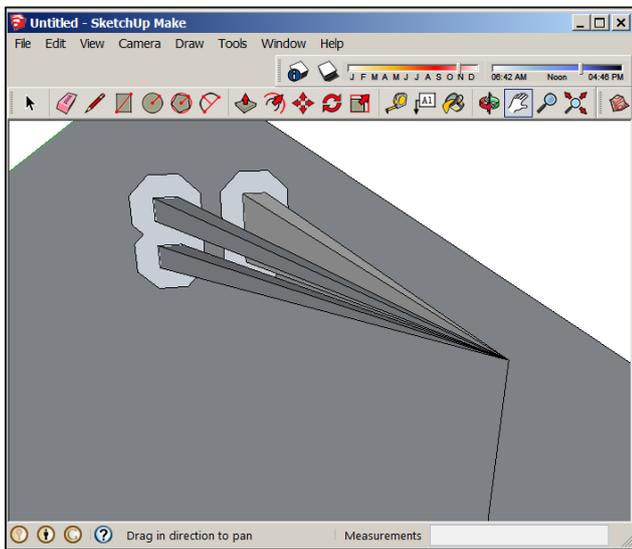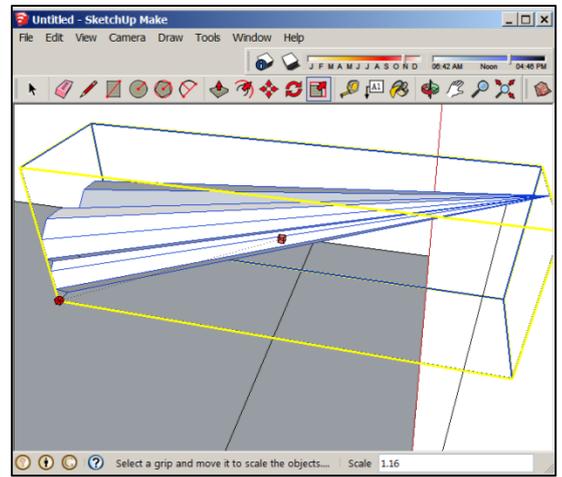

Group all of these triangles together (separately from the rest of the model).

Select the *Scale* tool and resize **from the corner while holding down *Ctrl*** by any amount. Holding down *Ctrl* ensures that it resizes by the same amount in every direction.
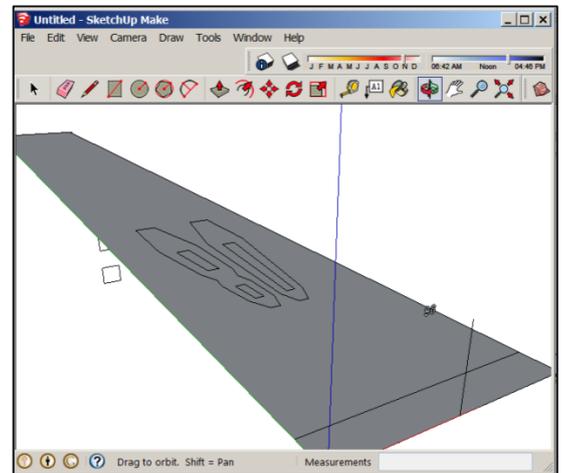
Immediately type *20* and press *Enter* to make the selection 20 times as big.

You will notice that the triangles are no longer positioned correctly. Reposition them so they all start at the point representing the driver's eye.
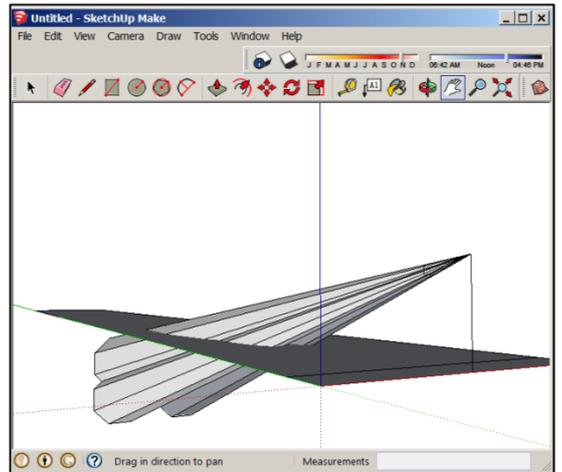
Right-click on the triangles and click *Intersect Faces > With Model*, then delete the triangles. The message should now appear on the road.

Ungroup everything, delete anything you do not need, and paint the message.

Note that in order to paint the message you may have to redraw the lines of the message over the top of the existing lines.