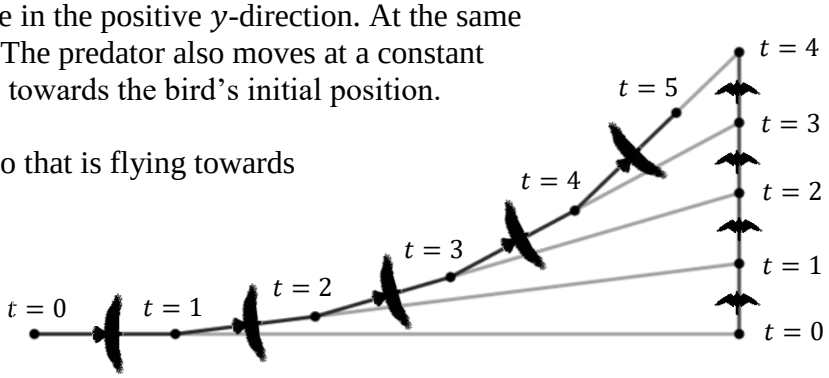
Motion Camouflage

A bird is flying a constant speed in a straight line in the positive y-direction. At the same time a predator leaves a tree and stalks the bird. The predator also moves at a constant speed, faster than the speed of the bird, and flies towards the bird's initial position.

The predator adjusts its direction every second so that is flying towards the bird's current position at that time.

When the predator is close enough to the bird, it strikes. The first few steps are shown in the diagram on the right.



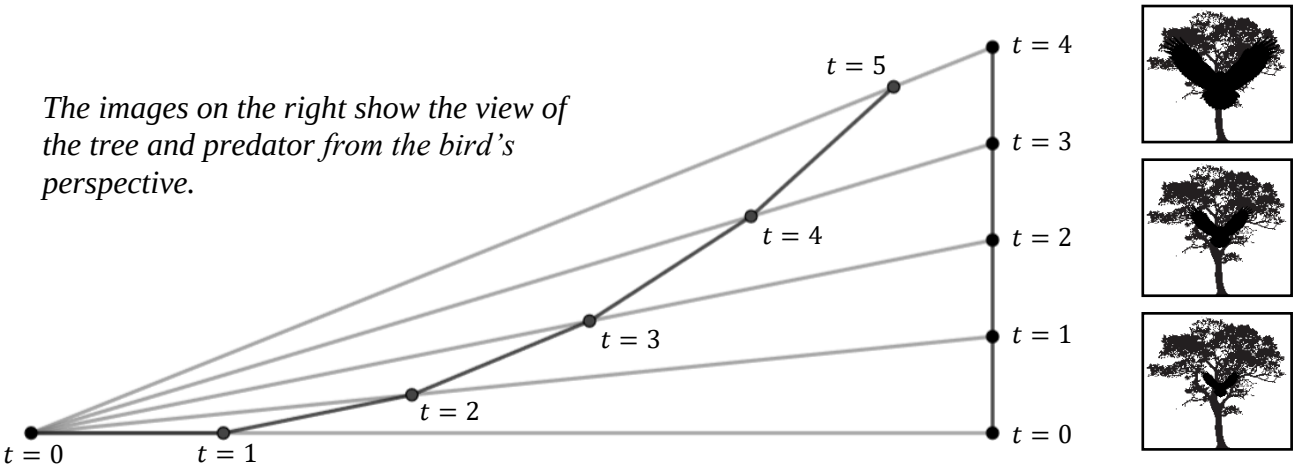
Task 1 (Levels 1 – 4 of criterion D and all levels of criterion C)

From the table on a later page determine initial positions for the bird and predator, and their speeds. Use trigonometry to investigate the method above to determine the approximate amount of time it takes for the predator to catch the bird.

You may assume the predator catches the bird when the distance between them (in metres) is less than the difference in their speeds (in m/s).

Next, instead of directly following the bird, the predator stalks the bird by staying on the bird's line-of-sight to the tree. This makes it harder for the bird to see the predator because from the bird's perspective the predator does not move in relation to its background (the tree). This is called *Motion Camouflage* and is demonstrated in the diagram below.

The images on the right show the view of the tree and predator from the bird's perspective.



Task 2 (Levels 1 – 6 of criterion D and all levels of criterion C)

Using the same values as in task 1 use trigonometry to investigate this new method and determine the approximate amount of time it takes for the predator to catch the bird.

Compare the advantages and disadvantages of the two methods of stalking the bird.

Task 3 (Levels 5 – 8 of criterion D and all levels of criterion C)

In the previous tasks the predator only updated its direction every second. Investigate what happens if its direction is updated more frequently. You will find it helpful to set up models in a spreadsheet.

Task 4 (All levels of criterion C and D)

Some accidents between cars and bicycles / motorbikes are caused by motion camouflage. Use what you have learned in this investigation to explain how this could occur in the following situations:

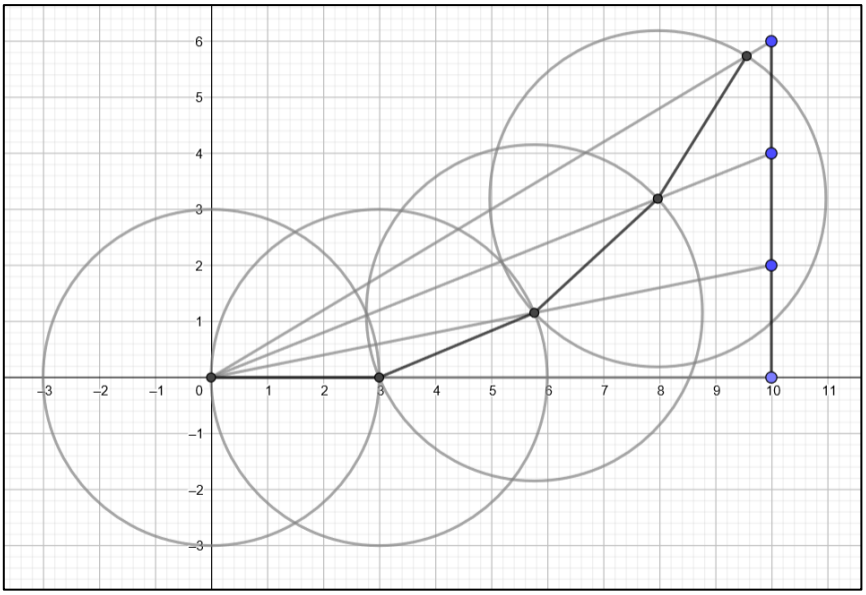
- The car and bicycle / motorbike are travelling in the same direction on the same road
- The car are bicycle / motorbike are travelling in opposite directions on opposite sides of the same road

Hints for creating diagrams

You could use Geogebra to create accurate diagrams. You should only need to use some of the following tools:

- Point
- Line
- Segment
- Circle: Center and Radius

Hide objects that do not need to be displayed in your final diagram.

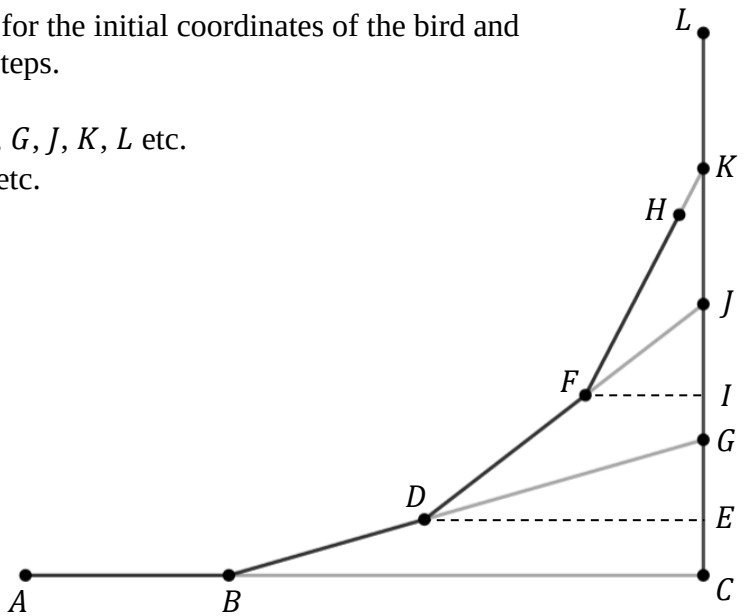


Hints for task 1

Use the diagram on the right, along with your values for the initial coordinates of the bird and Predator and their speeds to complete the following steps.

- Write down the coordinates of points A, B, C, G, J, K, L etc.
- Write down the lengths of AB, BD, DF, FH, etc.
- Determine the size of $\angle CBD$.
- Determine the coordinates of point D.
- Determine the size of $\angle EDF$.
- Determine the coordinates of point F.
- Determine the size of $\angle IFH$.
- Determine the coordinates of point H.
- etc.

After you have done the first few steps by hand you may find it easier to perform additional steps using a spreadsheet.



Name	Predator			Bird			Name	Predator			Bird		
	x	y	Speed	x	y	Speed		x	y	Speed	x	y	Speed
	0	0	3.9	20	0	2.9		0	0	2.2	10	0	1.5
	0	0	3.8	16	0	3		0	0	4.9	24	0	3.5
	0	0	5.1	30	0	3.7		0	0	4	20	0	2.7
	0	0	4.1	18	0	2.9		0	0	5.1	28	0	3.4
	0	0	2.6	12	0	1.8		0	0	3.5	16	0	2.4
	0	0	5	24	0	3.6		0	0	3.9	18	0	2.8
	0	0	2.5	14	0	1.7		0	0	3.8	18	0	2.5
	0	0	3.6	18	0	2.5		0	0	4.1	20	0	2.6
	0	0	4.4	22	0	2.9		0	0	2.9	14	0	2
	0	0	3.5	16	0	2.4		0	0	4.5	28	0	3.9
	0	0	2	10	0	1.4		0	0	3.8	18	0	2.5
	0	0	5.3	26	0	3.6		0	0	3.3	16	0	2.2

Criterion C: Communication in Mathematics		
Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below.	
1 – 2	The student is able to: I. use limited mathematical language II. use limited forms of mathematical representation to present information III. communicate through lines of reasoning that are difficult to interpret.	The student is able to: ○ attempt to explain the two methods of stalking the bird ○ create diagrams using a spreadsheet and/or Geogebra ○ attempt to explain conclusions
3 – 4	The student is able to: I. use some appropriate mathematical language II. use different forms of mathematical representation to present information adequately III. communicate through lines of reasoning that are able to be understood, although these are not always clear IV. adequately organize information using a logical structure.	The student is able to: ○ adequately explain the two methods of stalking the bird ○ create mostly accurate diagrams using a spreadsheet and/or Geogebra ○ adequately explain conclusions
5 – 6	The student is able to: I. usually use appropriate mathematical language II. usually use different forms of mathematical representation to present information correctly III. move between different forms of mathematical representation with some success IV. communicate through lines of reasoning that are clear although not always coherent or complete V. present work that is usually organized using a logical structure.	The student is able to: ○ clearly and concisely explain the two methods of stalking the bird ○ create accurate diagrams using a spreadsheet (and possibly) Geogebra ○ use the equation editor for all equations and expressions ○ create diagrams only containing details that are relevant to the model ○ clearly explain conclusions using results discovered in the investigation
7 – 8	The student is able to: I. consistently use appropriate mathematical language II. use different forms of mathematical representation to consistently present information correctly III. move effectively between different forms of mathematical representation IV. communicate through lines of reasoning that are complete and coherent V. present work that is consistently organized using a logical structure.	The student is able to: ○ clearly and concisely explain the two methods of stalking the bird using explanations and diagrams ○ create accurate diagrams using a spreadsheet (and possibly) Geogebra ○ use the equation editor for all equations and expressions ○ create diagrams only containing details that are relevant to the model ○ use appropriate linking sentences between explanations and diagrams (*the following diagram shows* etc.) ○ clearly explain conclusions using results discovered in the investigation

Criterion D: Applying Mathematics in Real Life Contexts		
Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below.	
1 – 2	The student is able to: I. identify some of the elements of the authentic real-life situation II. apply mathematical strategies to find a solution to the authentic real-life situation, with limited success.	The student is able to: ○ use trigonometry to determine some coordinates of the bird and predator in the two models
3 – 4	The student is able to: I. identify the relevant elements of the authentic real-life situation II. select, with some success, adequate mathematical strategies to model the authentic real-life situation III. apply mathematical strategies to reach a solution to the authentic real life situation IV. describe whether the solution makes sense in the context of the authentic real-life situation.	The student is able to: ○ use trigonometry to accurately determine coordinates of the bird and predator up to at least $t = 2$ in the two models ○ compare the advantages and disadvantages of the two models ○ explain why motion camouflage can sometimes cause accidents between cars and bicycles / motorbikes
5 – 6	The student is able to: I. identify the relevant elements of the authentic real-life situation II. select adequate mathematical strategies to model the authentic real-life situation III. apply the selected mathematical strategies to reach a valid solution to the authentic real-life situation IV. describe the degree of accuracy of the solution V. discuss whether the solution makes sense in the context of the authentic real-life situation.	The student is able to: ○ use trigonometry to accurately determine coordinates of the bird and predator up to at least $t = 3$ in the two models ○ create a spreadsheet to perform repeated calculations ○ thoroughly compare the advantages and disadvantages of the two models ○ investigate what happens when the predator's position is updated more frequently in both models ○ clearly explain why motion camouflage can sometimes cause accidents between cars and bicycles / motorbikes
7 – 8	The student is able to: I. identify the relevant elements of the authentic real-life situation II. select appropriate mathematical strategies to model the authentic real life situation III. apply the selected mathematical strategies to reach a correct solution IV. explain the degree of accuracy of the solution V. explain whether the solution makes sense in the context of the authentic real-life situation.	The student is able to: ○ use trigonometry to accurately determine coordinates of the bird and predator up to at least $t = 3$ in the two models ○ create a spreadsheet to perform repeated calculations ○ thoroughly compare the advantages and disadvantages of the two models ○ thoroughly investigate, using an appropriate spreadsheet, what happens when the predator's position is updated more frequently in both models ○ clearly explain, using accompanying diagrams, why motion camouflage can sometimes cause accidents between cars and bicycles / motorbikes