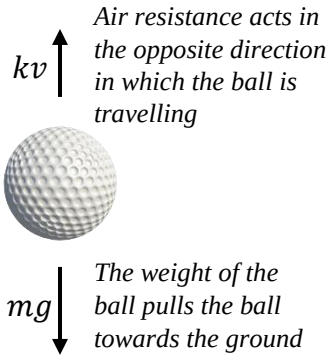
At Which Angle Does a Golf Ball Travel the Furthest?

A dropped object experiences two forces: the weight of the object, and air resistance.



The overall force F on the object is related to its mass m and acceleration a by the formula

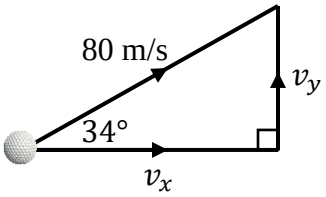
$$F = ma$$

The weight of an object is equal to mg where $g = 9.8$ ms^{-2}, the acceleration due to gravity. Let F_r be the force due to air resistance. We therefore have

$$F_r - mg = ma$$

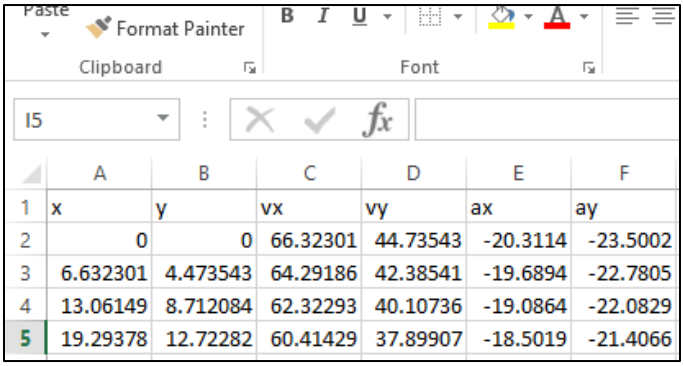
As the velocity v of an object increases so does the air resistance acting on it. Let $F_r = kv$ for some constant k.

- When $F_r = mg$ what can you say about the velocity of the golf ball?
- The terminal velocity of a 0.046 kg golf ball is 32 m/s. Use this to calculate the value of k.
- Hence determine an expression for the acceleration of a falling golf ball of mass 0.046 kg in terms of v.



We can apply this to two dimensions to investigate the trajectory of a golf ball. Suppose the golf ball has an initial speed of 80 m/s at an angle of 34° to the horizontal. This means its initial horizontal velocity v_x is equal to $80 \cos 34$ m/s and its initial vertical velocity v_y is equal to $80 \sin 34$ m/s.

The velocity of the ball, and therefore the acceleration, are both constantly changing. We can approximate the motion of the golf ball by assuming it moves at a constant velocity for a small time interval, calculating the new horizontal and vertical accelerations, using these to calculate the new horizontal and vertical velocities, and repeating the process. The screenshot on the right shows the position of the ball for the first four steps using a time interval of 0.1 seconds.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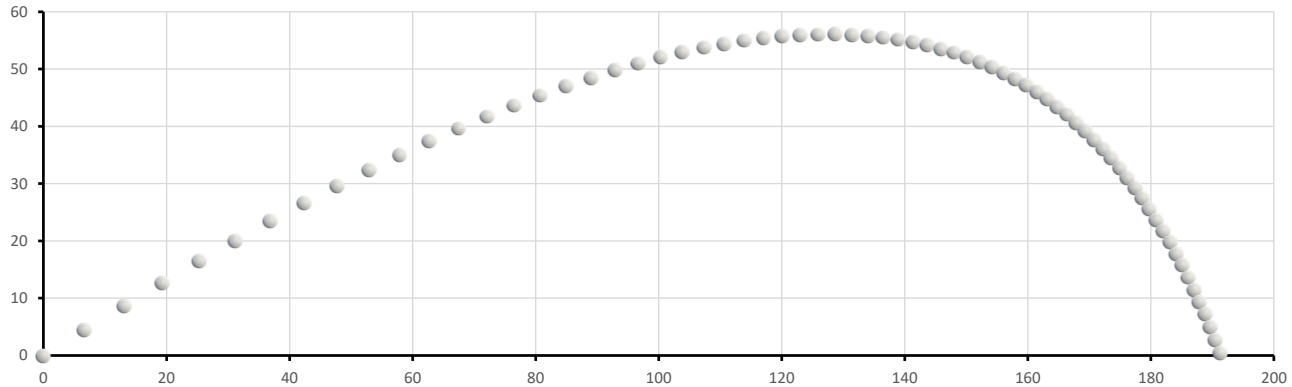
	A	B	C	D	E	F
1	x	y	vx	vy	ax	ay
2	0	0	66.32301	44.73543	-20.3114	-23.5002
3	6.632301	4.473543	64.29186	42.38541	-19.6894	-22.7805
4	13.06149	8.712084	62.32293	40.10736	-19.0864	-22.0829
5	19.29378	12.72282	60.41429	37.89907	-18.5019	-21.4066

The following graph shows the trajectory of the golf ball during its flight.



Use this as a starting point for an investigation. Extend it in any way you wish…

Criterion C: Communication in Mathematics		
Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below	
1 – 2	The student is able to: I. use limited mathematical language II. use limited forms of mathematical representation to present information III. communicate through lines of reasoning that are difficult to interpret.	The student is able to: o attempt to explain what is being investigated and how it is being investigated
3 – 4	The student is able to: I. use some appropriate mathematical language II. use appropriate forms of mathematical representation to present information adequately III. communicate through lines of reasoning that are complete IV. adequately organize information using a logical structure.	The student is able to o attempt to explain what is being investigated and how it is being investigated o use diagrams to justify some explanations and calculations o create a report that is able to be understood without referring to the task sheet
5 – 6	The student is able to: I. usually use appropriate mathematical language II. usually use appropriate forms of mathematical representation to present information correctly III. usually move between different forms of mathematical representation IV. communicate through lines of reasoning that are complete and coherent V. present work that is usually organized using a logical structure.	The student is able to o explain what is being investigated and how it is being investigated (including extending the given scenario) o use diagrams to justify explanations and calculations o move between explanations, calculations, tables and diagrams with appropriate linking sentences (the following table shows, figure 2 demonstrates etc.) o create a report that is able to be understood without referring to the task sheet
7 – 8	The student is able to: I. consistently use appropriate mathematical language II. use appropriate forms of mathematical representation to consistently present information correctly III. move effectively between different forms of mathematical representation IV. communicate through lines of reasoning that are complete, coherent and concise V. present work that is consistently organized using a logical structure	The student is able to o clearly explain what is being investigated and how it is being investigated (including extending the given scenario) o display formulae clearly and accurately using the equation editor o use clear and accurate diagrams to justify explanations and calculations o move effectively between explanations, calculations, tables and diagrams with appropriate linking sentences (the following table shows, figure 2 demonstrates etc.) o make good use of space on the page (no unnecessary white space, items positioned thoughtfully etc.) o create a report that is able to be understood without referring to the task sheet

Criterion D: Applying mathematics in real-life contexts		
Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below	
1 – 2	The student is able to: I. identify some of the elements of the authentic real-life situation II. apply mathematical strategies to find a solution to the authentic real-life situation, with limited success.	The student is able to o attempt to use appropriate mathematical tools and strategies to investigate the given scenario
3 – 4	The student is able to: I. identify the relevant elements of the authentic real-life situation II. select, with some success, adequate mathematical strategies to model the authentic real-life situation III. apply mathematical strategies to reach a solution to the authentic real-life situation IV. discuss whether the solution makes sense in the context of the authentic real-life situation.	The student is able to o use appropriate mathematical tools and strategies to investigate the given scenario
5 – 6	The student is able to: I. identify the relevant elements of the authentic real-life situation II. select adequate mathematical strategies to model the authentic real-life situation III. apply the selected mathematical strategies to reach a valid solution to the authentic real-life situation IV. explain the degree of accuracy of the solution V. explain whether the solution makes sense in the context of the authentic real-life situation.	The student is able to o use appropriate mathematical tools and strategies to accurately investigate the given scenario o give a good attempt at using appropriate mathematical tools and strategies to investigate an extension of the given scenario to a degree of rigour not below that of the given scenario
7 – 8	The student is able to: I. identify the relevant elements of the authentic real-life situation II. select appropriate mathematical strategies to model the authentic real-life situation III. apply the selected mathematical strategies to reach a correct solution to the authentic real-life situation IV. justify the degree of accuracy of the solution V. justify whether the solution makes sense in the context of the authentic real-life situation.	The student is able to o make thorough use of appropriate mathematical tools and strategies to accurately investigate the given scenario o make thorough use of appropriate mathematical tools and strategies to accurately investigate an extension of the given scenario to a degree of rigour not below that of the given scenario