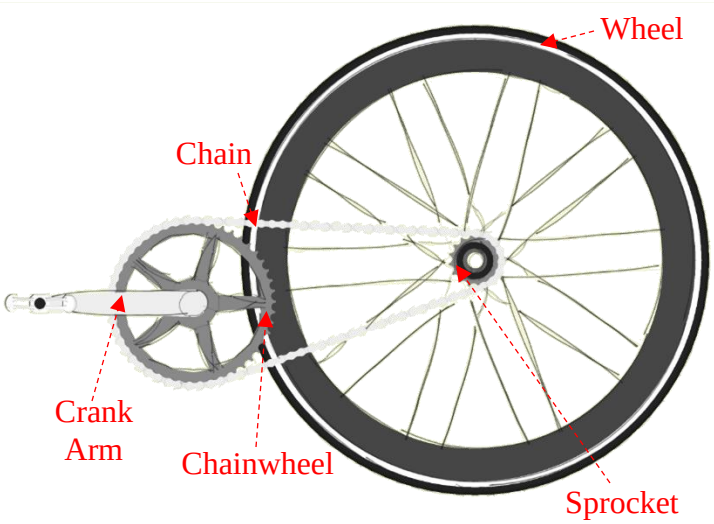
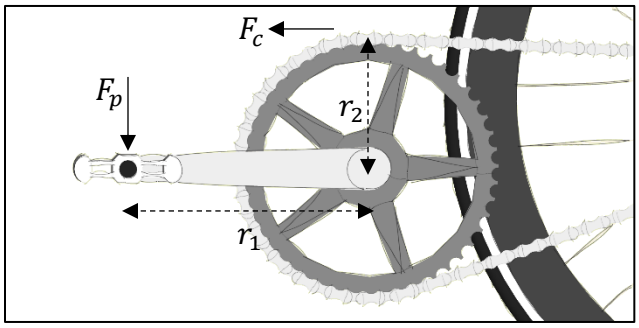
Designing a Race Winning Bicycle

Bicycles designed for racing on tracks have a single fixed gear. Consider the diagram on the right showing the parts of a bicycle that enable the rider to accelerate the bicycle forwards.



The rider exerts a force on the pedals. This exerts a force on the chain, which exerts a force on the sprocket, which is connected to the wheel, moving the bicycle forwards.

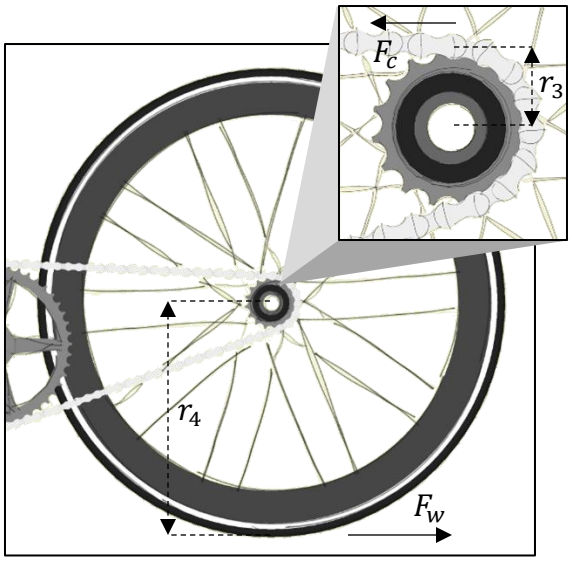
We can use a different sized chainwheel or sprocket to change how fast a bicycle accelerates, or change its top speed. The higher the top speed, the more difficult it is to accelerate.



Suppose the length of the crank arm is r_1 and the rider exerts a force on the pedal of F_p.

If the radius of the chainwheel is r_2 then the force exerted on the chain, F_c, will satisfy

$$F_p r_1 = F_c r_2$$



Suppose the radius of the sprocket is r_3. The chain exerts a force on the sprocket of F_c.

If the radius of the wheel is r_4 then the force exerted on the road by the wheel, F_w, will satisfy

$$F_c r_3 = F_w r_4$$

- Determine an expression for F_w in terms of F_p, r_1, r_2, r_3 and r_4.

The value of F_w is the size of the force that pushes the bicycle forwards.

The acceleration a of the bicycle will then satisfy the equation

$$F_w = ma$$

where m represents the total mass of the cyclist and the bicycle. Throughout this investigation we will assume that the value of m is equal to 70 kg.

There are three things which will limit the top speed of a cyclist: the amount of force he/she is able to exert on the pedal, his/her cadence (pedalling rate), and the air resistance acting on the cyclist and the bicycle.

Throughout this investigation we will assume the cyclist exerts a constant force of 700 N on the pedal until he/she reaches a maximum cadence of 150 revolutions per minute. In our initial model we will assume that air resistance is negligible. Let the length of the crank arm be 17 cm and the radius of the wheels be 35 cm.

Investigate sizes for the chainwheel and sprocket that enable the rider to complete a 2 km race in the fastest possible time.

Use this as a starting point for an investigation. Extend it in any way you wish…

Criterion C: Communication in Mathematics		
Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below	
1 – 2	The student is able to: I. use limited mathematical language II. use limited forms of mathematical representation to present information III. communicate through lines of reasoning that are difficult to interpret.	The student is able to: o attempt to explain what is being investigated and how it is being investigated
3 – 4	The student is able to: I. use some appropriate mathematical language II. use appropriate forms of mathematical representation to present information adequately III. communicate through lines of reasoning that are complete IV. adequately organize information using a logical structure.	The student is able to o attempt to explain what is being investigated and how it is being investigated o use diagrams to justify some explanations and calculations o create a report that is able to be understood without referring to the task sheet
5 – 6	The student is able to: I. usually use appropriate mathematical language II. usually use appropriate forms of mathematical representation to present information correctly III. usually move between different forms of mathematical representation IV. communicate through lines of reasoning that are complete and coherent V. present work that is usually organized using a logical structure.	The student is able to o explain what is being investigated and how it is being investigated (including extending the given scenario) o use diagrams to justify explanations and calculations o move between explanations, calculations, tables and diagrams with appropriate linking sentences (the following table shows, figure 2 demonstrates etc.) o create a report that is able to be understood without referring to the task sheet
7 – 8	The student is able to: I. consistently use appropriate mathematical language II. use appropriate forms of mathematical representation to consistently present information correctly III. move effectively between different forms of mathematical representation IV. communicate through lines of reasoning that are complete, coherent and concise V. present work that is consistently organized using a logical structure	The student is able to o clearly explain what is being investigated and how it is being investigated (including extending the given scenario) o display formulae clearly and accurately using the equation editor o use clear and accurate diagrams to justify explanations and calculations o move effectively between explanations, calculations, tables and diagrams with appropriate linking sentences (the following table shows, figure 2 demonstrates etc.) o make good use of space on the page (no unnecessary white space, items positioned thoughtfully etc.) o create a report that is able to be understood without referring to the task sheet

Criterion D: Applying mathematics in real-life contexts		
Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below	
1 – 2	The student is able to: I. identify some of the elements of the authentic real-life situation II. apply mathematical strategies to find a solution to the authentic real-life situation, with limited success.	The student is able to o attempt to use appropriate mathematical tools and strategies to investigate the given scenario
3 – 4	The student is able to: I. identify the relevant elements of the authentic real-life situation II. select, with some success, adequate mathematical strategies to model the authentic real-life situation III. apply mathematical strategies to reach a solution to the authentic real-life situation IV. discuss whether the solution makes sense in the context of the authentic real-life situation.	The student is able to o use appropriate mathematical tools and strategies to investigate the given scenario
5 – 6	The student is able to: I. identify the relevant elements of the authentic real-life situation II. select adequate mathematical strategies to model the authentic real-life situation III. apply the selected mathematical strategies to reach a valid solution to the authentic real-life situation IV. explain the degree of accuracy of the solution V. explain whether the solution makes sense in the context of the authentic real-life situation.	The student is able to o use appropriate mathematical tools and strategies to accurately investigate the given scenario o give a good attempt at using appropriate mathematical tools and strategies to investigate an extension of the given scenario to a degree of rigour not below that of the given scenario
7 – 8	The student is able to: I. identify the relevant elements of the authentic real-life situation II. select appropriate mathematical strategies to model the authentic real-life situation III. apply the selected mathematical strategies to reach a correct solution to the authentic real-life situation IV. justify the degree of accuracy of the solution V. justify whether the solution makes sense in the context of the authentic real-life situation.	The student is able to o make thorough use of appropriate mathematical tools and strategies to accurately investigate the given scenario o make thorough use of appropriate mathematical tools and strategies to accurately investigate an extension of the given scenario to a degree of rigour not below that of the given scenario