# How to Draw the Perfect Sketch

## Introduction

By having an artist as my father, I have been continuously exposed to sketches and portraits. The main idea of drawing a sketch would be to depict the chosen object as perfectly and as precisely as possible. Artists take years to master this technique, and even with years of experience, drawing a perfect sketch still proves to be difficult.

However, if a mathematical approach is taken to draw a perfect sketch, this may lead to a better quality sketch. Therefore, this exploration aims to identify methods of drawing the perfect sketch through the use of mathematics.

## Method 1: Introducing the Situation

When sketching an object, for example a cube, it is necessary to transform the 3-dimensional cube into a 2-dimensional shape. In order to represent this visually, a 2-dimensional diagram was constructed through GeoGebra:

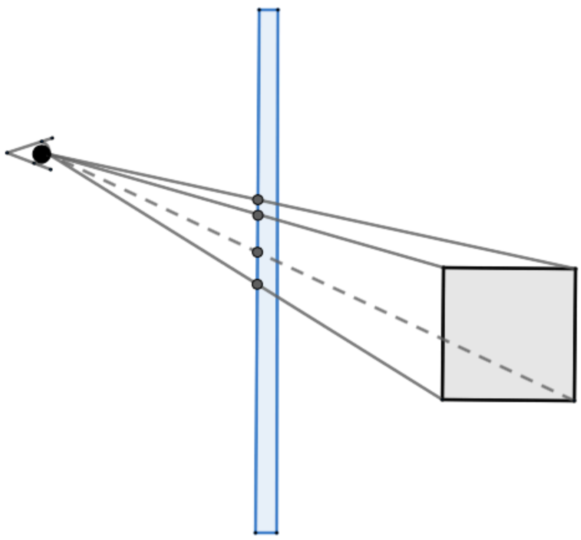

*Diagram 1*

In Diagram 1, the essential aspects of the situation are introduced. The eye on the left side of the diagram represents the artist's eye. In the center of the diagram, there is a blue rectangle, which represents the artist's canvas or paper on which the artist will draw the sketch. The lines connecting the vertices to the eye represents the artist's view. By looking at the object through the canvas, this allows accurate coordinates to be calculated through mathematical means, and it is a realistic situation for any artist, since artists sketch objects that are usually in front of them.

To enhance the understanding of the situation and mathematically represent the situation realistically, the following model is a 3-dimensional model was constructed. The canvas is represented by the black outline on the *x*-*z* plane and the vertices of the cube are red to make the diagram easier to view.

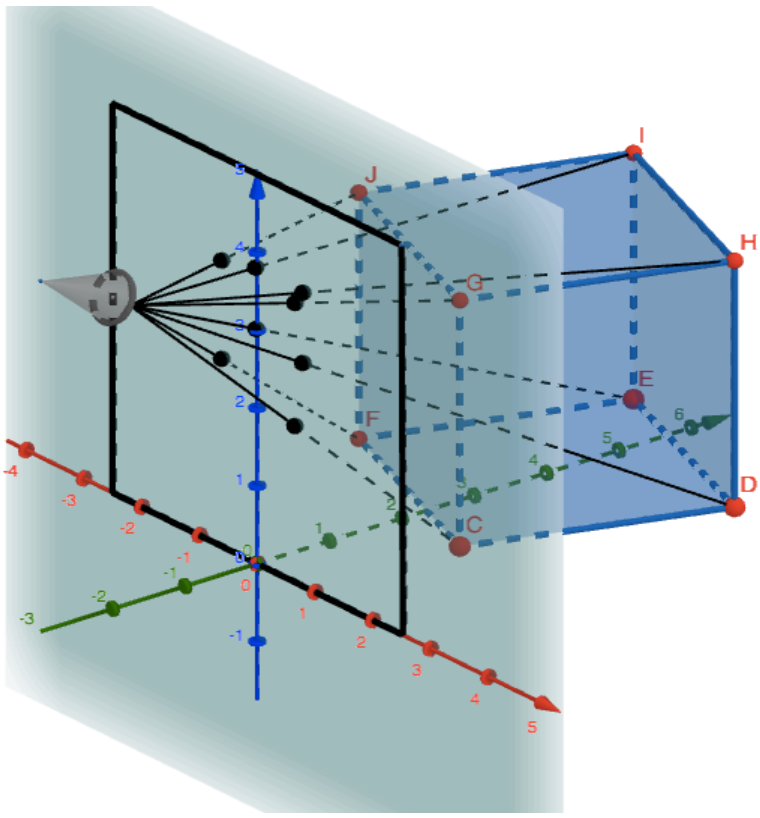


*Diagram 2*

The following are the coordinates of the vertices of the cube and the eye, which were chosen randomly. When initially analyzing the diagram, and by anticipating the result through reflecting, I made the cube have a slight angle, which emphasizes the realistic situation artists face when drawing sketches.

Eye: (0, -2, 4)

Vertex C: (1, 2, 0)

Vertex D: (2, 5, 0)

Vertex E: (-1, 6, 0)

Vertex F: (-2, 3, 0)

Vertex G: (1, 2, $\sqrt{10}$)

Vertex H: (2, 5, $\sqrt{10}$)

Vertex I: (-1, 6, $\sqrt{10}$)

Vertex J: (-2, 3, $\sqrt{10}$)

The height of the cube ($\sqrt{10}\ units$), was calculated using the Pythagoras Theorem. The following is an example calculation for Vertex G:

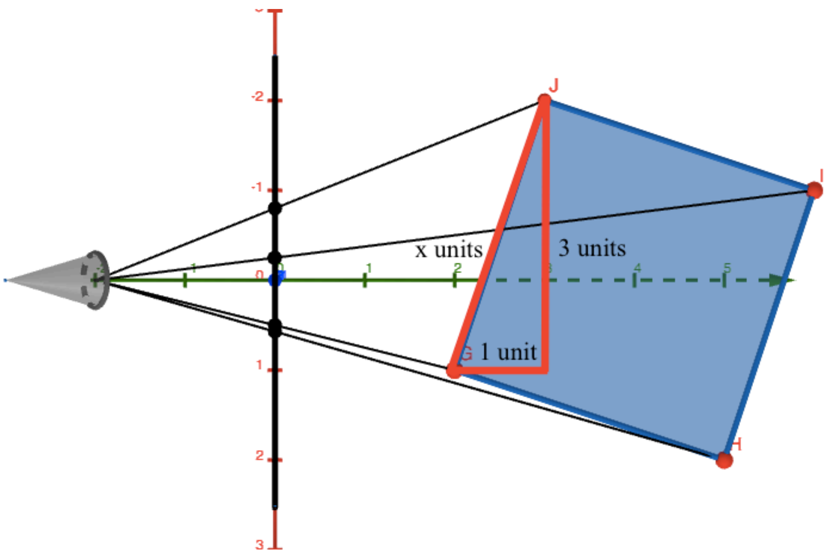


*Diagram 3*

$$a^2 + b^2 = c^2$$
$$1^2 + 3^2 = c^2$$
$$c = \pm\sqrt{10}$$

The canvas is represented with this equation, since the canvas is placed on the *x-z* plane:

$$y = 0$$

By clarifying the setting of the exploration, example calculations can now be conducted in order to find the coordinates of the vertices. For this method, the standard equation for the symmetric equation of the line will be used. Although this is not part of the IB syllabus, through reviewing its function, this equation proved to be a potential method in solving the problem. (Dawkins) The following calculations will be an example calculation for Vertex C in *Diagram 2*.

$$\frac{x-x_0}{a}=\frac{y-y_0}{b}=\frac{z-z_0}{c}$$

In this equation, $(x_0, y_0, z_0)$ represents the point in which the line passes through. Respectively, this will be the coordinate of the artist's eye, since the artist will only be able to see the object through his/her eye. Since the eye's coordinate is (0, -2, 4), that will be substituted into the equation.

$$\frac{x-0}{a}=\frac{y+2}{b}=\frac{z-4}{c}$$

Then, the coordinates of Vertex C, (1, 2, 0) will be substituted into the equation to determine (*a, b, c*).

$$\frac{1-0}{a}=\frac{2+2}{b}=\frac{0-4}{c}$$

$$\frac{1}{a}=\frac{4}{b}=\frac{-4}{c}$$

After this is established, one value can be substituted freely, since the coordinates will be in 2-dimensions, which leaves one extra variable within (*a, b, c*). Here, *a* was replaced by 1:

$$\frac{1}{1}=\frac{4}{b}=\frac{-4}{c}$$

$$1=\frac{4}{4}=\frac{-4}{-4}$$

Therefore, $a=1,\ \ b=4,\ \ c=-4$ . Now, by replacing (*a, b, c*) into the original symmetric equation of the line with the coordinate of the artist's eye, it is possible to solve for (*x, y, z*), which represent the coordinate on the plain.

$$\frac{x-0}{a}=\frac{y+2}{b}=\frac{z-4}{c}$$

$$\frac{x-0}{1}=\frac{y+2}{4}=\frac{z-4}{-4}$$

Here, since there are three unknown variables, this would prove to be unsolvable. However, since it was previously explained that $y=0$, that can be substituted in.

$$\frac{x-0}{1}=\frac{0+2}{4}=\frac{z-4}{-4}$$

$$x=\frac{1}{2}=\frac{z-4}{-4}$$

Therefore, $x=\frac{1}{2}$, and $y=0$.

$$\frac{1}{2}=\frac{z-4}{-4}$$

$$-2=z-4$$

$$z=2$$

Therefore, the coordinates of Vertex C on the 2-dimensional canvas is ($\frac{1}{2}$, 0, 2).

The next example calculation will feature the same methodology as the aforementioned example, which will be Vertex J, $(-2, 3, \sqrt{10})$. This vertex was chosen to demonstrate calculations with the height of $\sqrt{10}$:

$$\frac{x-0}{a}=\frac{y+2}{b}=\frac{z-4}{c}$$

$$\frac{-2-0}{a}=\frac{3+2}{b}=\frac{\sqrt{10}-4}{c}$$

In this example, *a* will be substituted with $-2$.

$$\frac{-2}{-2}=\frac{5}{b}=\frac{\sqrt{10}-4}{c}$$

$$\frac{-2}{-2}=\frac{5}{5}=\frac{\sqrt{10}-4}{\sqrt{10}-4}$$

Therefore, $a=-2$, $b=5$, $c=\sqrt{10}-4$.

$$\frac{x-0}{a}=\frac{y+2}{b}=\frac{z-4}{c}$$

$$\frac{x-0}{-2}=\frac{y+2}{5}=\frac{z-4}{\sqrt{10}-4}$$

Then, 0 will substitute *y*.

$$\frac{x}{-2} = \frac{2}{5} = \frac{z-4}{\sqrt{10}-4}$$

$$\frac{x}{-2} = \frac{2}{5}$$

$$x = -\frac{4}{5}$$

$$\frac{2}{5} = \frac{z-4}{\sqrt{10}-4}$$

$$2(\sqrt{10}-4) = 5(z-4)$$

$$2\sqrt{10}-8 = 5z-20$$

$$2\sqrt{10}+12 = 5z$$

$$z = \frac{2\sqrt{10}+12}{5}$$

Therefore, the coordinates of Vertex J on the 2-dimensional canvas is $(-\frac{4}{5}, 0, \frac{2\sqrt{10}+12}{5})$.

By completing this process for every vertex of the object, the coordinate of every vertex was calculated. Then, by entering the coordinates into GeoGebra, the following sketch was produced:

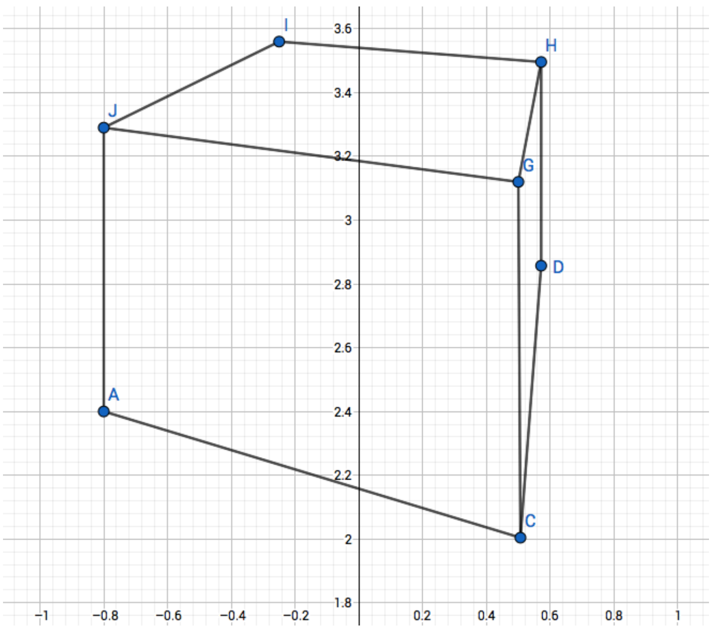


*Diagram 4*

By calculating the coordinate of the vertices through algebraic functions and equations, the perfect sketch was produced.

Method 2: Further Mathematical Application

After reflecting on the previous example with the cube, the methodology in solving the problem could be considered a limitation. Since the process was long, this increases the chance of making careless errors. Therefore, another mathematical method was considered, which is the use of vectors. When using vectors, it is necessary to label the vertices, which is done in the diagram below. The canvas is represented by the black outline on the *x*-*z* plane and the vertices of the cube are red to make the diagram easier to view.

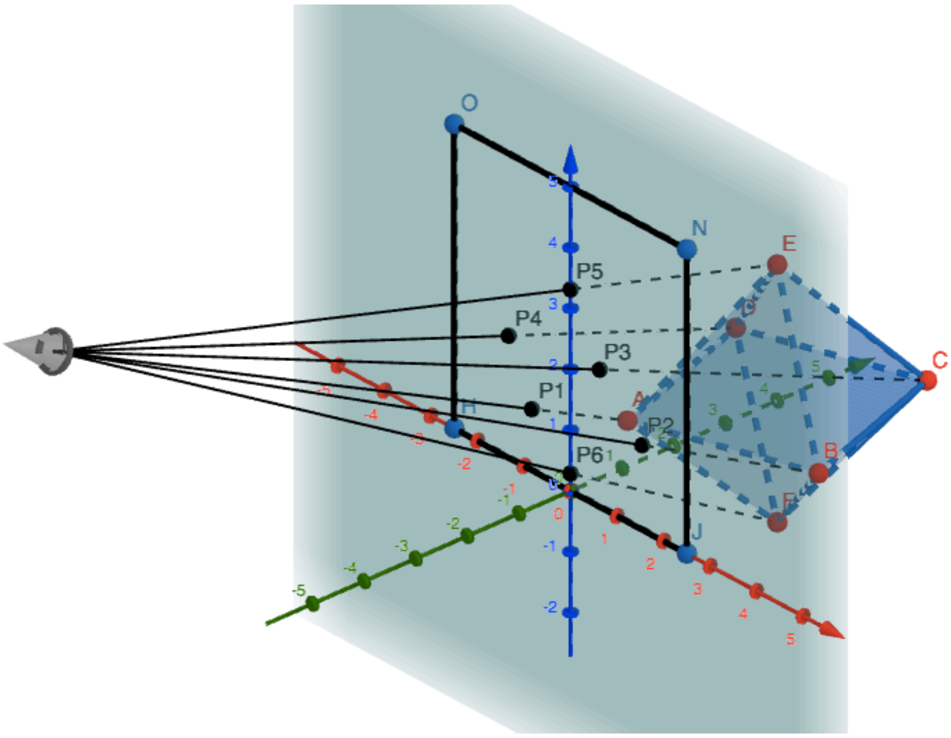


*Diagram 5*

Since artists draw multiple shapes, which may be more complex than cubes, an octahedron was used as the second example. Again, the coordinates of the vertices were randomly chosen:

Eye (I): (0, -10, 6)

Vertex A: (-1, 2, 0)

Vertex B: (2, 3, 0)

Vertex C: (1, 6, 0)

Vertex D: (-2, 5, 0)

Vertex E: $(0, 4, \sqrt{5})$

Vertex F: $(0, 4, -\sqrt{5})$

Determining the vector equations and parametric equations of lines can be used when calculating 3-dimensional figures with the use of this equation, where O is the origin, P is the canvas on $y$=0 and I is the eye. The bold letters represent the vector:

$$\mathbf{OP} = \mathbf{OI} + t(\mathbf{IA})$$

Since the coordinate for I is (0, -10, 6), this can be written as position vector **OI**, which is $\begin{pmatrix} 0 \\ -10 \\ 6 \end{pmatrix}$. As part of the example calculation, Vertex A will be used. Now that vector **OI** has been identified, this equation can be set up:

$$\mathbf{OP}_1 = \begin{pmatrix} 0 \\ -10 \\ 6 \end{pmatrix} + t(\mathbf{IA})$$

In order to determine **IA**, the following method will be used.

$$\mathbf{IA} = \mathbf{OA} - \mathbf{OI}$$

$$\mathbf{IA} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ -10 \\ 6 \end{pmatrix}$$

$$\mathbf{IA} = \begin{pmatrix} -1 \\ 12 \\ -6 \end{pmatrix}$$

Therefore, the following equation is determined.

$$\mathbf{OP}_1 = \begin{pmatrix} 0 \\ -10 \\ 6 \end{pmatrix} + t\begin{pmatrix} -1 \\ 12 \\ -6 \end{pmatrix}$$

$$\mathbf{OP}_1 = \begin{pmatrix} -t \\ 12t - 10 \\ 6 - 6t \end{pmatrix}$$

Since it is already determined that $y = 0$, the following can be substituted.

$$12t - 10 = 0$$

$$t = \frac{5}{6}$$

By determining the parameter $t$, the result can be substituted back into the original equation:

$$\mathbf{OP_1} = \begin{pmatrix} -\frac{5}{6} \\ 0 \\ 6 - 6(\frac{5}{6}) \end{pmatrix}$$

$$\mathbf{OP_1} = \begin{pmatrix} -\frac{5}{6} \\ 0 \\ 1 \end{pmatrix}$$

This position vector acts as the coordinate of point $P_1$ on *Diagram 5*, which is $(-\frac{5}{6}, 0, 1)$.

After calculating all of the coordinates of the vertices, the coordinates were put into GeoGebra, which produced this perfect sketch of an octahedron.

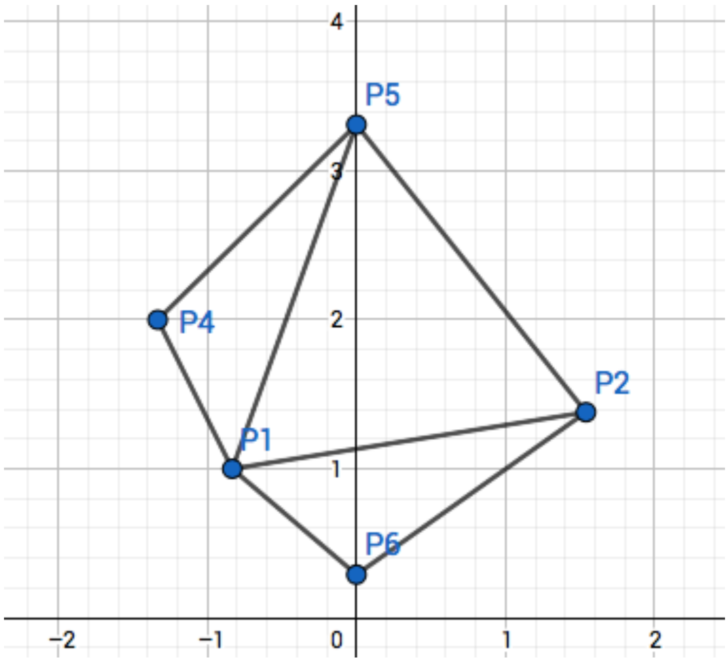


*Diagram 6*

Conclusion

By completing two sketches of two different objects with two different mathematical approaches, this proves how calculating coordinates and plotting them on a graph allows perfect sketches to be made. As *Diagram 4* and *Diagram 6* prove, by applying math to the field of the arts, a perfectly drawn cube or octahedron was created. Also, by not including certain vertices, such as Vertex P3 for *Diagram 6*, it adds to the realism of the sketch, since artists cannot see through the object, unless it is transparent.

By analyzing the two methods and determining whether it is a useful approach in drawing the perfect sketch, the answer would be yes, since graphic design companies, or other online entrepreneurs can use this technology to create more complex sketches.

Also, by doing this exploration, it allows people to understand how sketching and figuring out coordinates can be done in various ways, and artists could be able to use this in the fields of architecture, making this exploration beneficial, not only to those who have access to an online program, but in real life situations as well. Therefore, this exploration would like to conclude that it is possible to draw perfect sketches.

Works Cited

Dawkins, Paul. "Equations of Lines." *Equations of Lines*. 2003.